Bounded-Independence Sampling of Edges for Combinatorial Graph Properties

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Edge Subsampling in Graphs

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- introducing cycle-freeness under subsampling

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Useful for parallel "matroid basis finding" in "(co)graphic matroids"
[Karp, Upfal, and Wigderson '85, '88]
[Khanna, Putterman, and Song '25]

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Bounded Independence

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A distribution $X = (x_1, ..., x_n)$ is δ-almost k-wise independent with marginals p if $\forall S \subseteq [n]$, $|S| \leq k$

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Theorem [Naor and Naor '90]

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all match the behavior of $\mathcal{G}(n, p)$.

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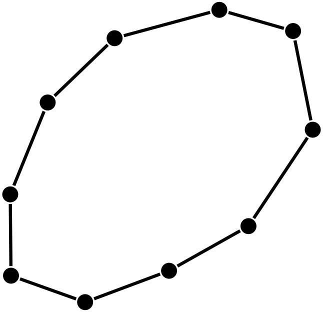

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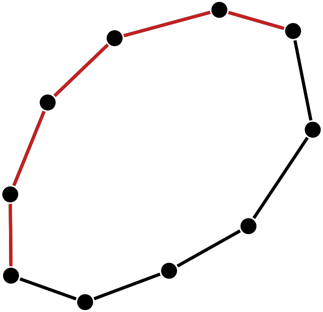

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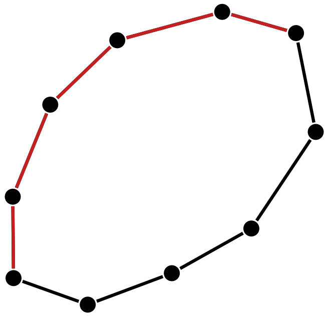

Then, every cycle must contain a path of length $9 \log m$.
Hence, it suffices to show that no such path survives sampling.

Cycle-freeness: fully independent sampling

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$$1 - \left(\frac{1}{m^9} \cdot 2m^2\right) = 1 - \frac{2}{m^7} = 1 - \frac{1}{\mathrm{poly}(m)}$$

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$$\left(\frac{1}{2}\right)^{9\log m} + \delta = \left(\frac{1}{2}\right)^{9\log m} + \frac{1}{m^9} = \frac{2}{m^9}$$

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Spectral Sparsification via Bounded Independence Sampling

Theorem [Doron, Murtagh, Vadhan, and Zuckerman '20] (Derandomization of [Spielman and Srivastava '11])

In a weighted graph G, subsampling the edges with

$$\textit{marginals } p_e = \Theta\left(\ell(e) \cdot \log(m)\right),$$

where $\ell(e)$ is the leverage score of e, in a $O(\log m)$-wise independent manner produces a "spectral sparsifier" (and hence preserves connected components) of G with probability $1 - 1/\operatorname{poly}(m)$.

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Corollary

If leverage scores are bounded by $1/\kappa \log(m)$ for large enough constant κ, $O(\log m)$-wise independent sampling with marginals $1/2$ preserves connected components.

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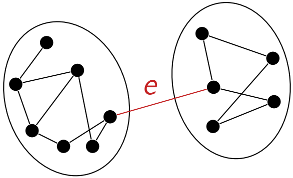
Then the leverage score of an edge $e \in E$ is defined as

$$\ell(e) := \Pr[e \in T].$$

Leverage Scores



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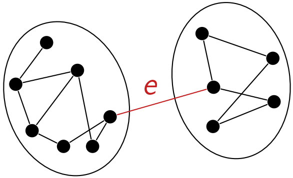

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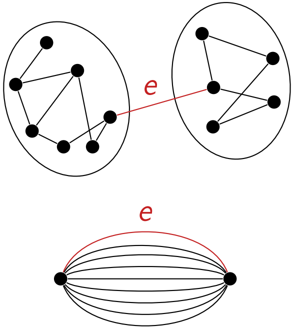

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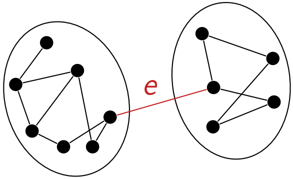

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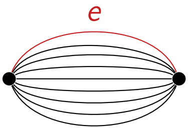

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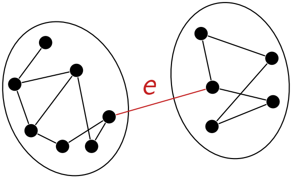

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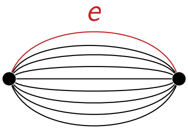

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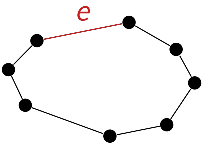

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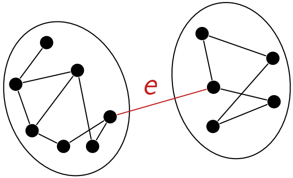

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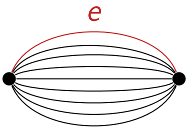

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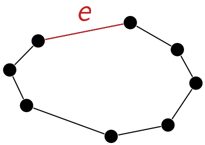

$\ell(e) = 1 - 1/m$

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Want to show:
If the minimum cut is at least $\kappa \log(m)$, $O(\log m)$-wise independent sampling with marginals $1/2$ preserves connectedness.

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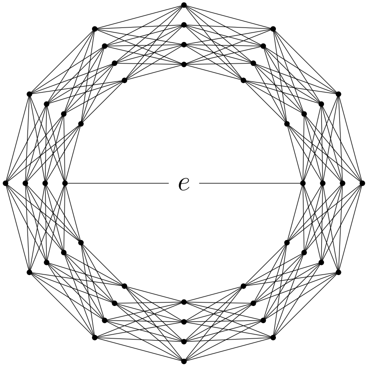
Minimum Cut vs Leverage Scores

Minimum cut can be $\Omega(\log n)$ while $\ell(e) \to 1$:



Getting Low Leverage Scores

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Can we find a weighting of G which makes leverage scores small?

Leverage Scores & Minimum Cut Duality

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For any $G = (V, E)$ with minimum cut c, there exists a weighting $w : E \to \mathbb{R}^+$ such that in the weighted graph $G' = (V, E, w)$, all leverage scores are bounded by $O(1/c)$.

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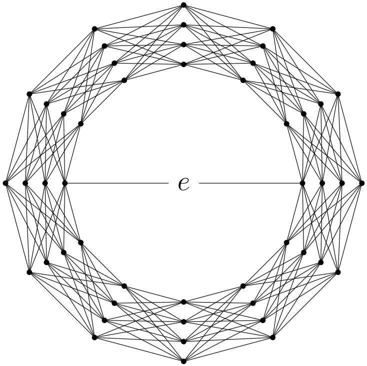
min cut $c \implies$ leverage scores $O(1/c)$ under some reweighting

Leverage Scores & Minimum Cut Duality

Decreasing the weight of e ensures the bound on leverage scores.



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Thanks!