Condensing and Extracting Against Online Adversaries




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Background

Randomness is Useful

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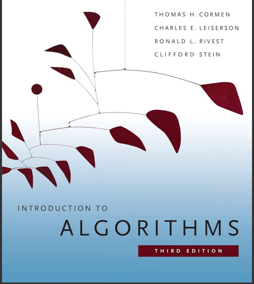

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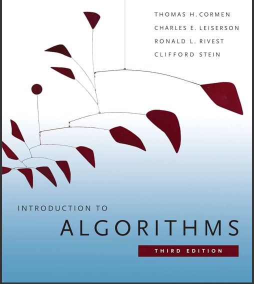

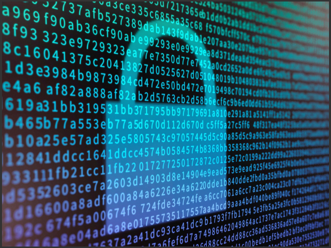

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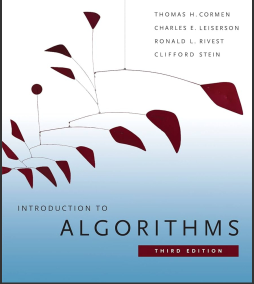

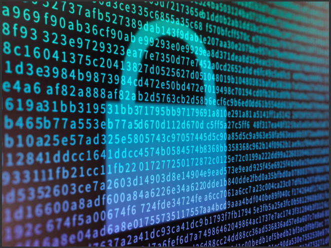

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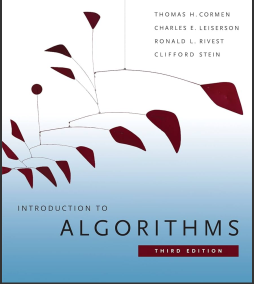

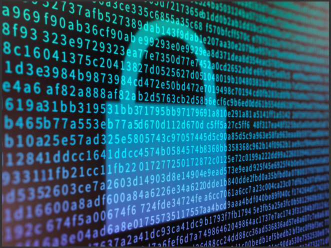

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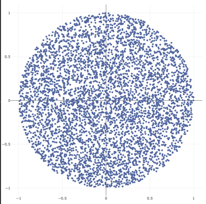

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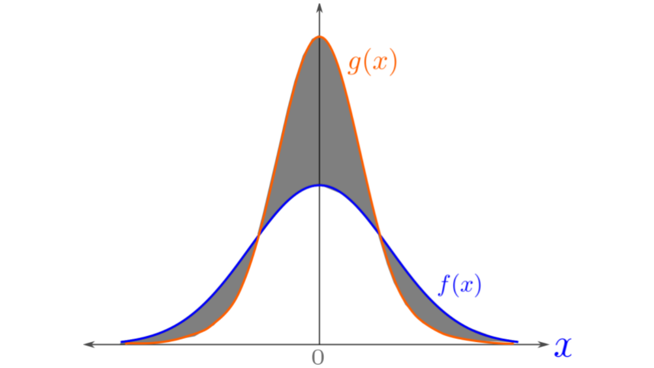
$$|\mathbf{A} - \mathbf{B}| = \max_{H \subset \{0,1\}^r} |\Pr[\mathbf{A} \in H] - \Pr[\mathbf{B} \in H]| = \frac{1}{2} \cdot |\mathbf{A} - \mathbf{B}|_1$$



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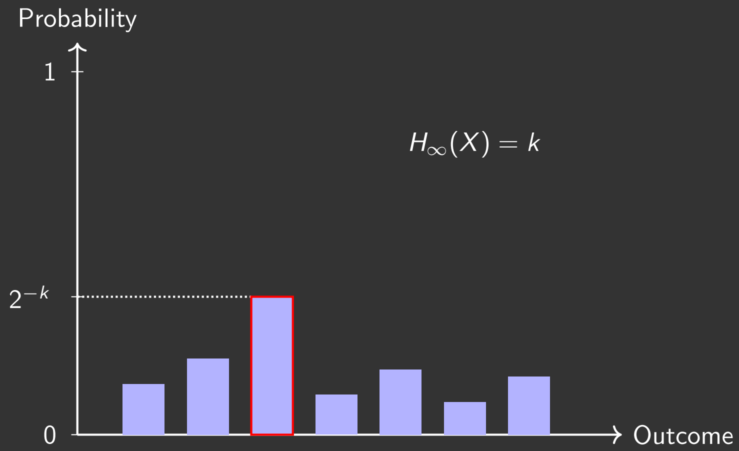

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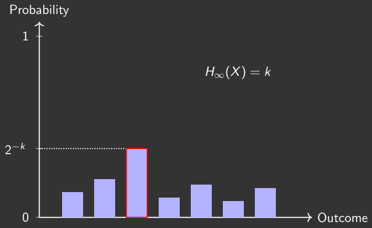

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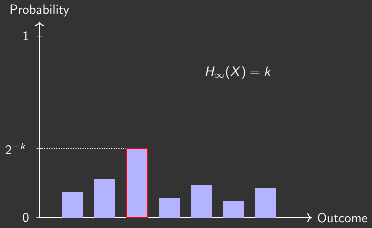

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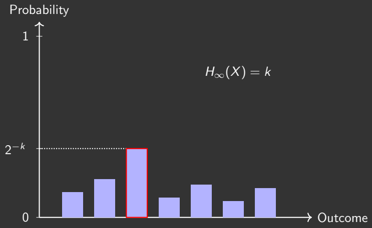

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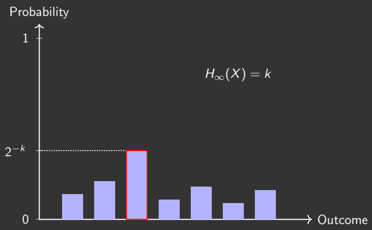

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Smooth Min-Entropy

For $\varepsilon \geq 0$, $H^\varepsilon_\infty(\mathbf{X}) = \max_{|\mathbf{X}-\mathbf{Y}| \leq \varepsilon} H_\infty(\mathbf{Y})$

Randomness Purification - Condensing and Extracting

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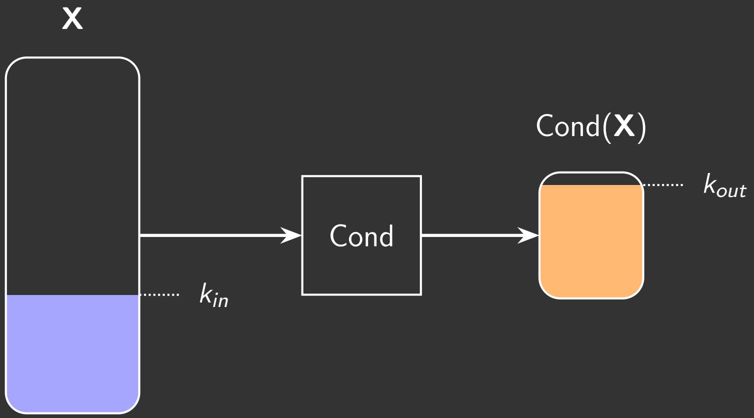

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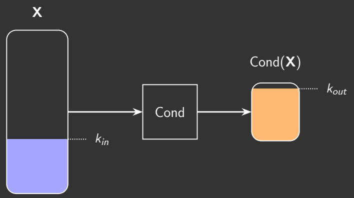

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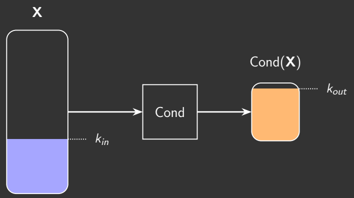

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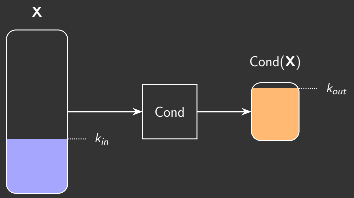

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Extractor

Extractor is condenser with $\Delta_{out} = 0$, $\frac{k_{out}}{m} = 1$

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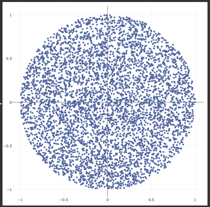

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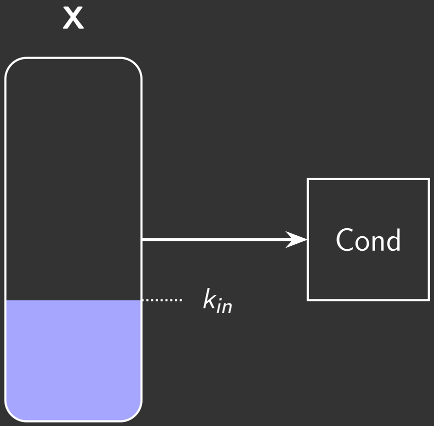

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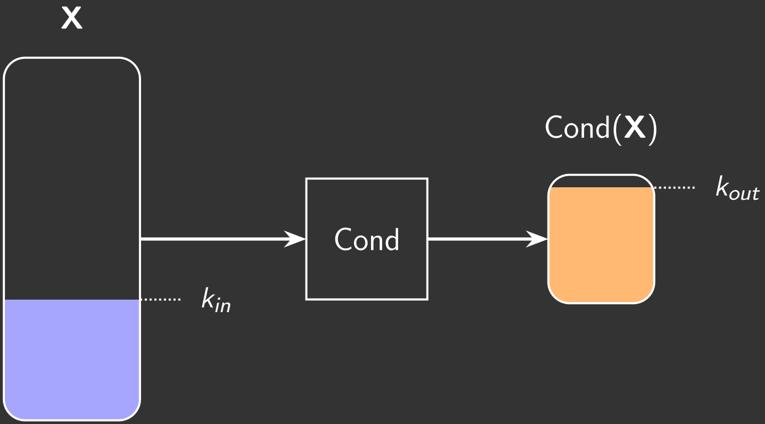

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- "One-shot simulations" for randomized protocols, cryptography, interactive proofs etc.

oNOSF sources

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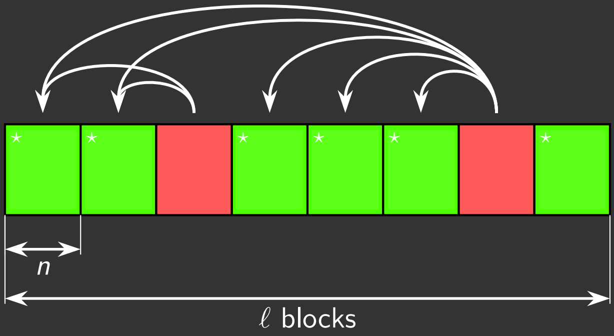

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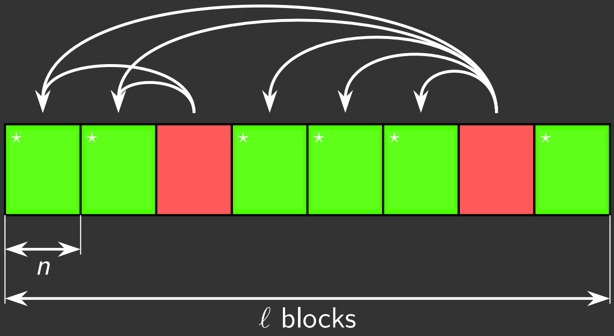

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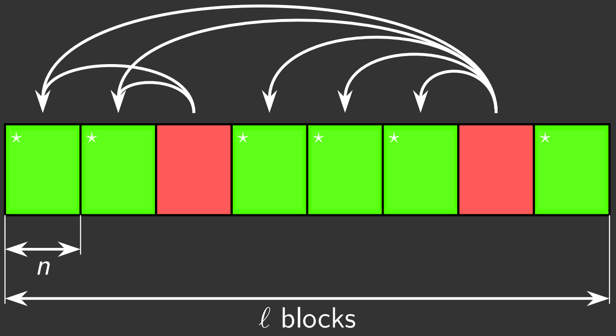

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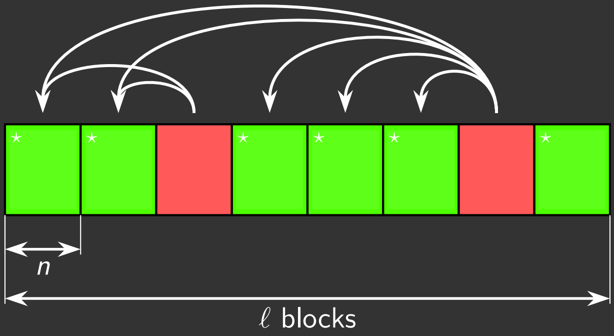

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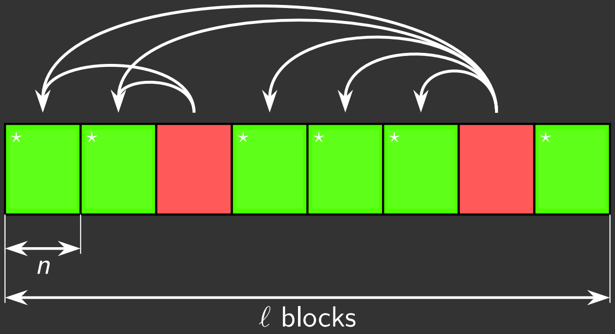

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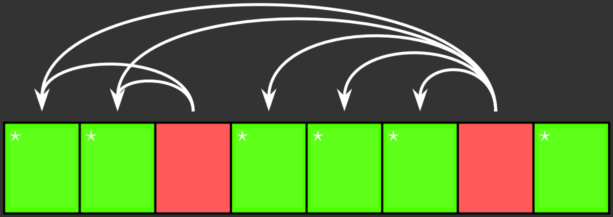

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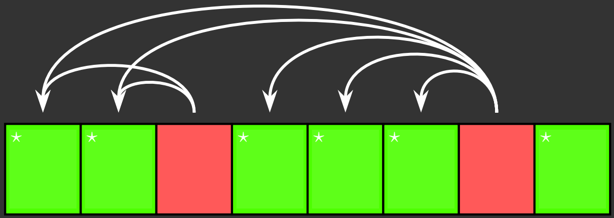
Blockchains as randomness beacon - CRS model



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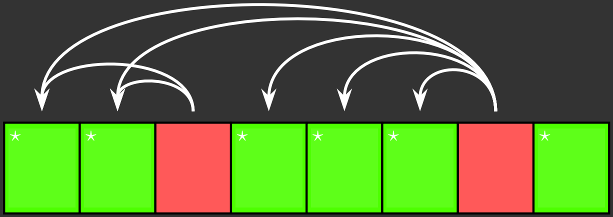

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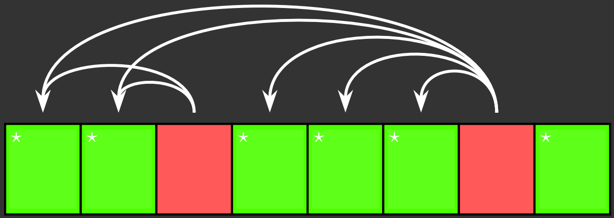

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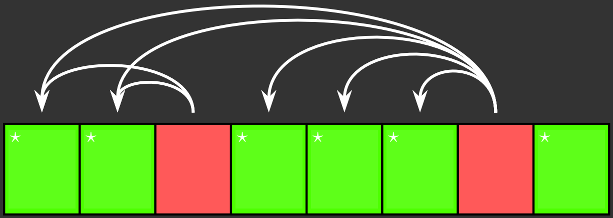

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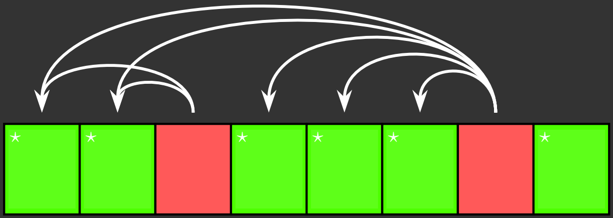

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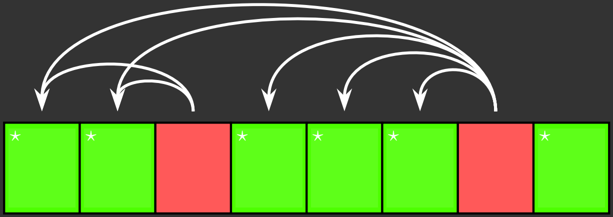

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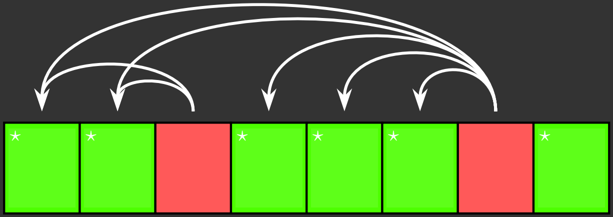
- Take turns broadcasting their sources



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Chattopadhyay – G – Ringach (CGR)'24

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Condensing against Online Adversaries

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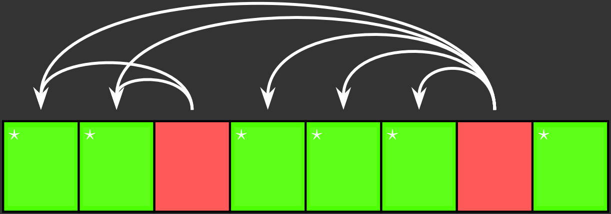

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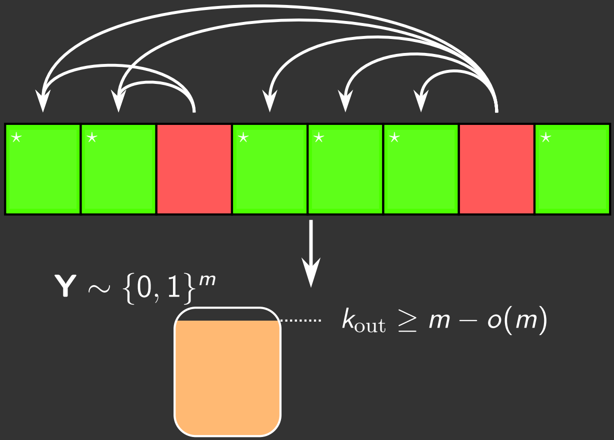

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CGR'24 : For any $f : (\{0,1\}^n)^\ell \to \{0,1\}^m$, exists $(\boldsymbol{g = 0.5\ell}, \ell, n)$ oNOSF source $\mathbf{X}$ s.t. $\boldsymbol{H_\infty^\varepsilon(f(\mathbf{X})) \leq 0.5m}$, $\varepsilon = 0.1$

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CGR'24 : For non-online sources, can't condense when $\boldsymbol{g = (1-\delta)\ell}$, beyond **rate $\boldsymbol{1-\delta}$**

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Why not $g \leq 0.5\ell$

CGR'24 : For any $f : (\{0,1\}^n)^\ell \to \{0,1\}^m$, exists $(\boldsymbol{g = 0.5\ell}, \ell, n)$ oNOSF source $\mathbf{X}$ s.t. $\boldsymbol{H_\infty^\varepsilon(f(\mathbf{X})) \leq 0.5m}$, $\varepsilon = 0.1$

Why not non-online

CGR'24 : For non-online sources, can't condense when $\boldsymbol{g = (1-\delta)\ell}$, beyond **rate** $\boldsymbol{1 - \delta}$

Why not extract uniform randomness?

Condensing from oNOSF Sources

Input: (g, ℓ, n) oNOSF source with $\boldsymbol{g \geq 0.51\ell}$
Output: $\mathbf{Y} \sim \{0,1\}^m$ with $\boldsymbol{H_\infty^\varepsilon(\mathbf{Y}) = k_{out} \geq m - o(m)}$

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AORSV'20 : Can't extract even if $g = 0.99\ell$

Condensing from oNOSF Sources - Constructions

Input: (g, ℓ, n) oNOSF source $\mathbf{X}$ with $\boldsymbol{g \geq 0.51\ell}$
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Condensing from oNOSF Sources - Constructions

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Output: $\mathbf{Y} \sim \{0,1\}^m$ with $\boldsymbol{H_\infty^\varepsilon(\mathbf{Y}) \geq m - o(m)}$
Parameters: $\boldsymbol{m = O(\ell n), \varepsilon = 1/\mathrm{poly}(n)}$

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Problem gets **harder** as $\boldsymbol{n}$ gets **smaller**

Work	Existential / Explicit	n

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Work	Existential / Explicit	n
CGR'24	Existential	$\geq 2^{\Omega(\ell)}$

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Our Result	Existential	$\geq \omega(1)$

Condensing from oNOSF Sources - Constructions

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Output: $\mathbf{Y} \sim \{0,1\}^m$ with $H_\infty^\varepsilon(\mathbf{Y}) \geq m - o(m)$
Parameters: $m = O(\ell n), \varepsilon = 1/\text{poly}(n)$

Problem gets **harder** as n gets **smaller**

Work	Existential / Explicit	n
CGR'24	Existential	$\geq 2^{\Omega(\ell)}$
Our Result	Existential	$\geq \omega(1)$
Our Result	Explicit	$\geq \text{polylog}(\ell)$

Condensing from entropic and $g = 1$ oNOSF source

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Statement

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- $\mathbf{A} \sim \{0,1\}^r$ s.t. $H_\infty(\mathbf{A}) \geq 0.01r$

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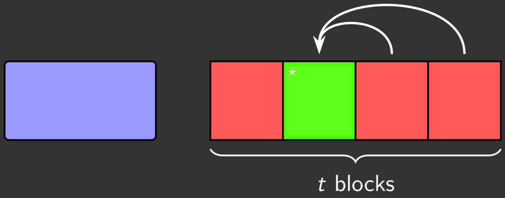
- $\mathbf{B}$: $(1, t, n)$ oNOSF source with $n \geq 2^{\Omega(t)} \cdot \log(r/\varepsilon)$



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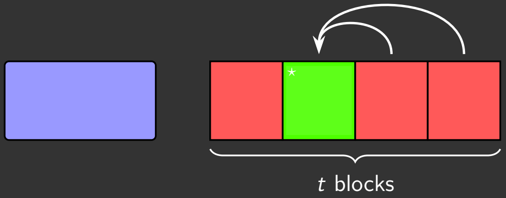

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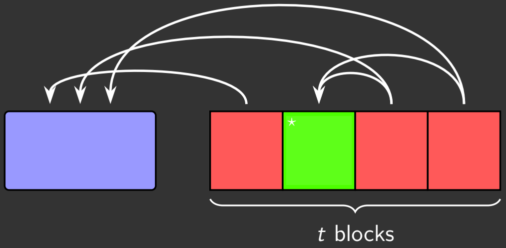
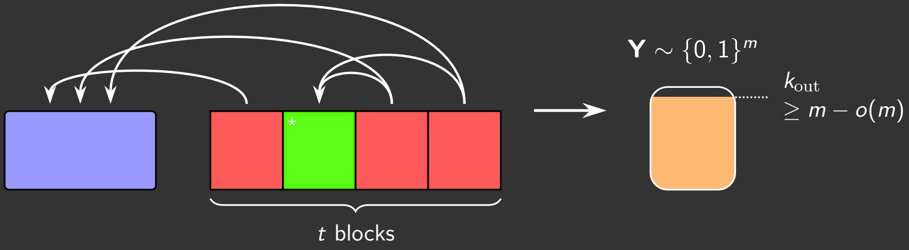


Condensing from entropic and $g = 1$ oNOSF source

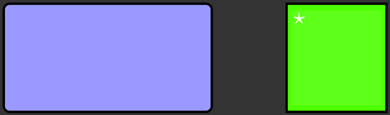
Statement

- $\mathbf{A} \sim \{0,1\}^r$ s.t. $H_\infty(\mathbf{A}) \geq 0.01r$
- $\mathbf{B}$: $(1, t, n)$ oNOSF source with $n \geq 2^{\Omega(t)} \cdot \log(r/\varepsilon)$
- Good block in **B** **independent** of **A**
- Bad blocks in **B** **depend arbitrarily** on **A**
- Can **explicitly** construct Cond s.t.
  $H^{\varepsilon}_{\infty}(\mathrm{Cond}(\mathbf{A}, \mathbf{B})) \geq m - o(m)$ with $m = \Omega(r)$



Condensing from entropic and $g = 1$ oNOSF source

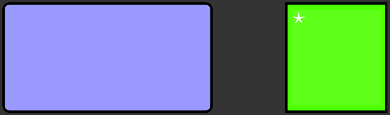
What if $g = t = 1$



Condensing from entropic and $g = 1$ oNOSF source

What if $g = t = 1$

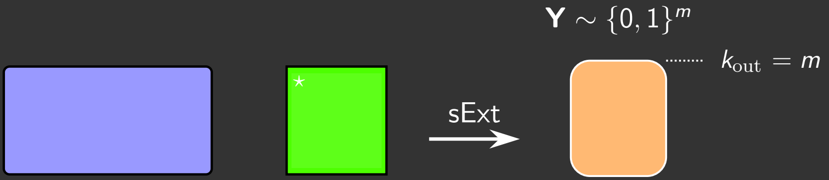
- **A**: **Entropic**, **B**: **independent** and **uniform**



Condensing from entropic and $g = 1$ oNOSF source

What if $g = t = 1$

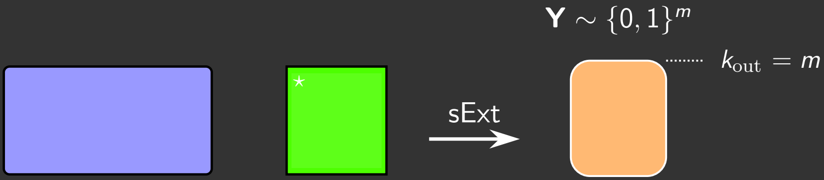
- **A**: **Entropic**, **B**: **independent** and **uniform**
- Exactly need **seeded extractors** Nisan-Zuckerman'96



Condensing from entropic and $g = 1$ oNOSF source

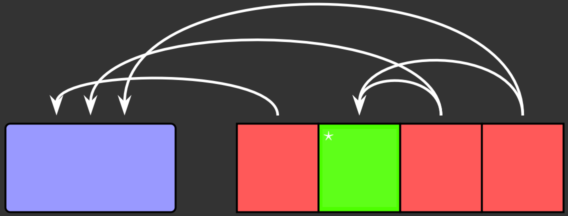
What if $g = t = 1$

- **A**: **Entropic**, **B**: **independent** and **uniform**
- Exactly need **seeded extractors** Nisan-Zuckerman'96
- Output **sExt(A, B)**



Condensing from entropic and $g = 1$ oNOSF source

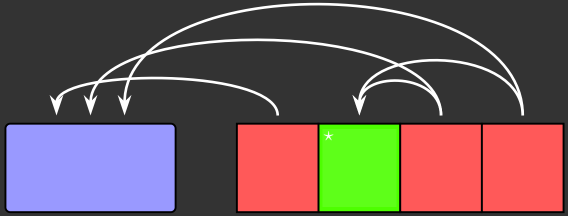
For $g = 1, t > 1$



Condensing from entropic and $g = 1$ oNOSF source

For $g = 1, t > 1$

- **A**: **Entropic**, **B**: **$(1, t, n)$ oNOSF** source, good index j



Condensing from entropic and $g = 1$ oNOSF source

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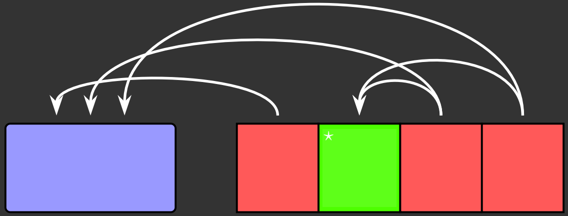

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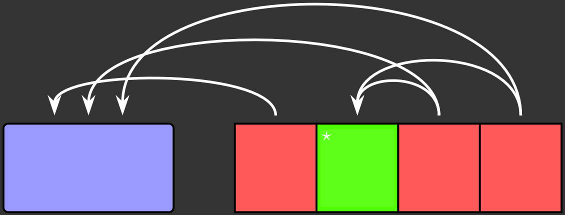

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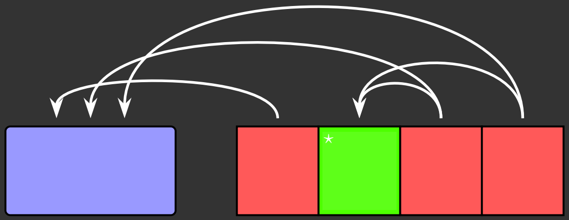
- So what?



Condensing from entropic and $g = 1$ oNOSF source

For $g = 1, t > 1$ and Last block good

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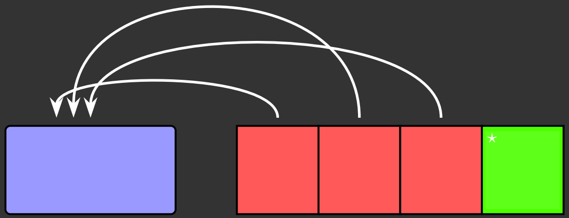

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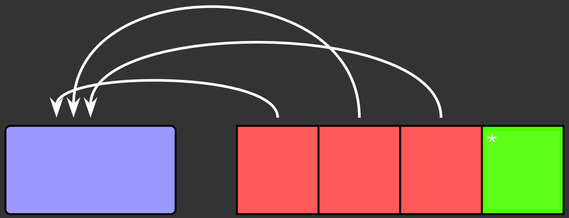
Proof of Uniform output



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Proof of Uniform output

1. Condition on fixing bad blocks - $\mathbf{B}_1 = b_1, \ldots, \mathbf{B}_{t-1} = b_{t-1}$

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   - **A**: **Entropic**, $\mathbf{B}_t$: **Uniform and Independent**

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Condensing from entropic and $g = 1$ oNOSF source

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3. By property of **sExt**, Output — **Uniform**

Condensing from entropic and $g = 1$ oNOSF source

For $g = 1, t > 1$ and First block good

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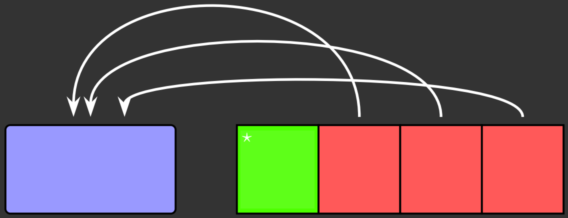

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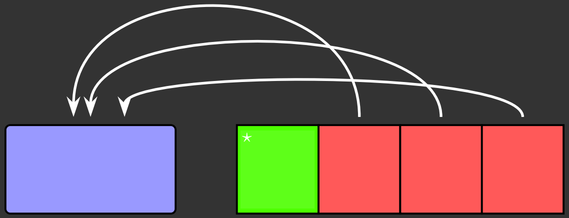
Adversary can't make things too bad lemma CGR'24



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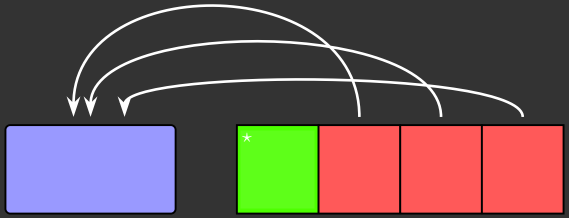
Say $\mathbf{Y} = f(\mathbf{X})$ and b bits in **X** **controlled by adversary**



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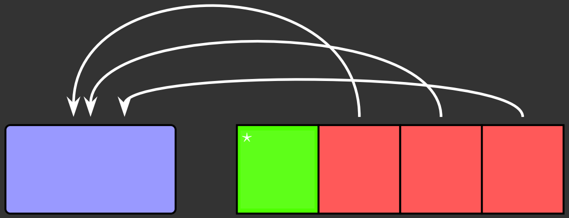

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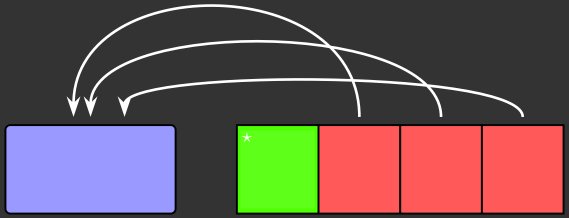
Then, $\mathbf{Y}$ s.t. $H_\infty^{\varepsilon \cdot 2^b}(\mathbf{Y}) \geq k_{out} - b$



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Applying above

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Applying above

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Applying above

- Adversary controls $b = n \cdot (t - 1)$ bits
- If all blocks uniform, $k_{out} = m$

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For $g = 1, t > 1$ and First block good

- **A**: **Entropic**, **B**: $(1, t, n)$ **oNOSF** source, good index $j = 1$
- Construction: $\bigoplus_{i=1}^{t} \mathsf{sExt}(\mathbf{A}, \mathbf{B}_i)$

Adversary can't make things too bad lemma CGR'24

Say $\mathbf{Y} = f(\mathbf{X})$ and b bits in $\mathbf{X}$ **controlled by adversary**
If those b bits **uniform**, $\mathbf{Y}_{\text{uniform}}$ s.t. $H_\infty^{\varepsilon}(\mathbf{Y}_{\text{uniform}}) \geq k_{out}$
Then, $\mathbf{Y}$ s.t. $H_\infty^{\varepsilon \cdot 2^b}(\mathbf{Y}) \geq k_{out} - b$

Applying above

- Adversary controls $b = n \cdot (t - 1)$ bits
- If all blocks uniform, $k_{out} = m$
- Can ensure $k_{out} - b \geq m - o(m)$

Condensing from entropic and $g = 1$ oNOSF source

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- Need $\varepsilon < 2^{-b}$

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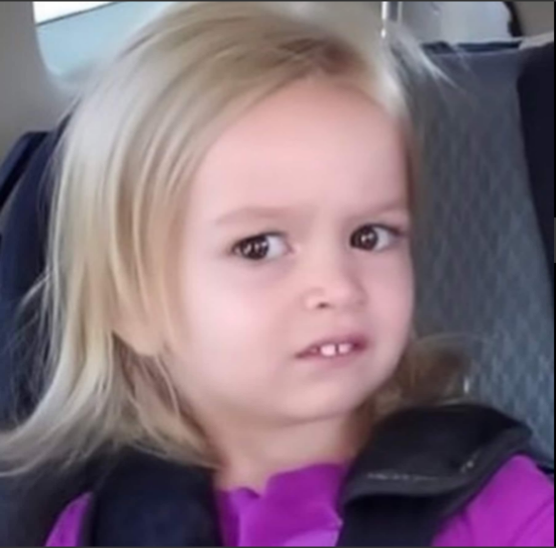
- Need $\varepsilon < 2^{-b}$
  By sExt requirements, $n \geq 2\log(1/\varepsilon) \geq 2b = 2n \cdot (t - 1)$



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Condensing from entropic and $g = 1$ oNOSF source

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Applying Adversary can't make things too bad lemma

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Applying Adversary can't make things too bad lemma

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- Need $\varepsilon < 2^{-b}$, and by sExt requirements,
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Condensing from entropic and $g = 1$ oNOSF source

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- Key observation: This works if $|\mathsf{B}_1| > 2\sum_{i=2}^{t} |\mathsf{B}_t|$

Condensing from entropic and $g = 1$ oNOSF source

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For $g = 1, t > 1$ and position j good

Condensing from entropic and $g = 1$ oNOSF source

For $g = 1, t > 1$ and First block good

- **A**: **Entropic**, **B**: $(\mathbf{1}, \boldsymbol{t}, \boldsymbol{n})$ **oNOSF** source, good index $\boldsymbol{j} = \mathbf{1}$
- Construction: $\bigoplus_{i=1}^{t} \mathsf{sExt}(\mathbf{A}, \mathbf{B}_i)$

Applying Adversary can't make things too bad lemma

- Need $\varepsilon < 2^{-b}$, and by sExt requirements,
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- Key observation: This works if $|\mathbf{B}_1| > 2\sum_{i=2}^{t} |\mathbf{B}_t|$

For $g = 1, t > 1$ and position j good

- Require $|\mathbf{B}_j| > 2\sum_{i=j+1}^{t} |\mathbf{B}_t|$

Condensing from entropic and $g = 1$ oNOSF source

For $g = 1, t > 1$ and First block good

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Applying Adversary can't make things too bad lemma

- Need $\varepsilon < 2^{-b}$, and by sExt requirements,
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- Key observation: This works if $|\mathbf{B}_1| > 2\sum_{i=2}^{t} |\mathbf{B}_t|$

For $g = 1, t > 1$ and position j good

- Require $|\mathbf{B}_j| > 2\sum_{i=j+1}^{t} |\mathbf{B}_t|$
- Works if $|\mathbf{B}_i| = 2^{\Omega(t-i)}$

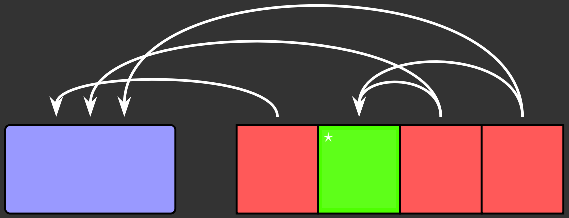
Condensing from entropic and $g = 1$ oNOSF source

Actual Construction

Condensing from entropic and $g = 1$ oNOSF source

Actual Construction

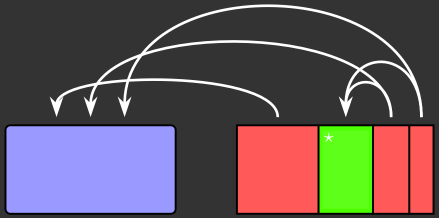
- **A**: **Entropic**, **B**: **$(1, t, n)$ oNOSF** source



Condensing from entropic and $g = 1$ oNOSF source

Actual Construction

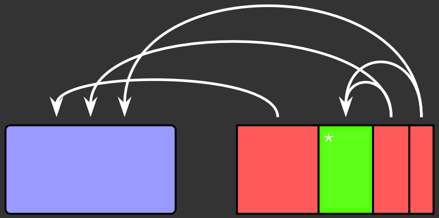
- **A**: **Entropic**, **B**: **$(1, t, n)$ oNOSF** source
- Let $\mathbf{B}'_i$ be **first $2^{\Omega(t-i)}$** bits of $\mathbf{B}_i$



Condensing from entropic and $g = 1$ oNOSF source

Actual Construction

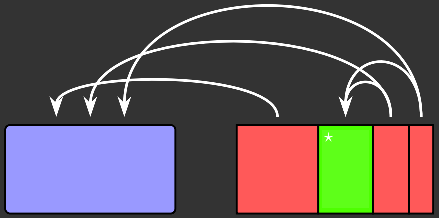
- **A**: **Entropic**, **B**: **$(1, t, n)$ oNOSF** source
- Let **B'_i** be **first $2^{\Omega(t-i)}$** bits of **B**$_i$
- Construction: $\bigoplus_{i=1}^{t} \mathsf{sExt}(A, B'_i)$



Condensing from entropic and $g = 1$ oNOSF source

Actual Construction

- **A**: **Entropic**, **B**: $\boldsymbol{(1, t, n)}$ **oNOSF** source
- Let $\mathbf{B}'_i$ be **first** $\mathbf{2^{\Omega(t-i)}}$ bits of $\mathbf{B}_i$
- Construction: $\bigoplus_{i=1}^{t} \mathbf{sExt}(\mathbf{A}, \mathbf{B}'_i)$
- Output entropy $\boldsymbol{m - o(m)}$



Condensing from oNOSF Sources

Condensing from entropic and $g = 1$ oNOSF source

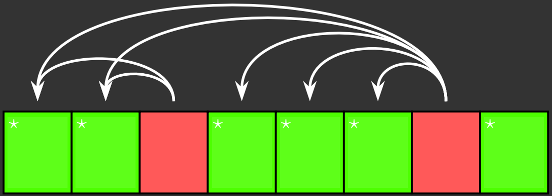
A: entropic, **B**: $(g = 1, t, n = 2^{\Omega(t)})$ oNOSF

Condensing from oNOSF Sources

Condensing from entropic and $g = 1$ oNOSF source

A: entropic, **B**: $(g = 1, t, n = 2^{\Omega(t)})$ oNOSF

Condensing from $g = 0.51\ell$, $n = 2^{\Omega(\ell)}$ oNOSF source



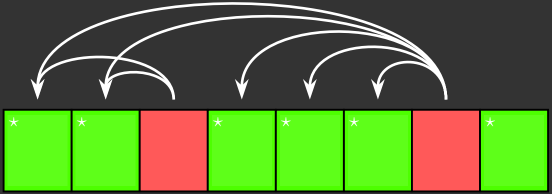
Condensing from oNOSF Sources

Condensing from entropic and $g = 1$ oNOSF source

A: entropic, **B**: $(\boldsymbol{g = 1, t, n = 2^{\Omega(t)}})$ oNOSF

Condensing from $g = 0.51\ell$, $\boldsymbol{n = 2^{\Omega(\ell)}}$ oNOSF source

- Given $\boldsymbol{g = 0.51\ell}$ oNOSF source $\mathbf{X} = (\mathbf{X}_1, \ldots, \mathbf{X}_\ell)$



Condensing from oNOSF Sources

Condensing from entropic and $g = 1$ oNOSF source

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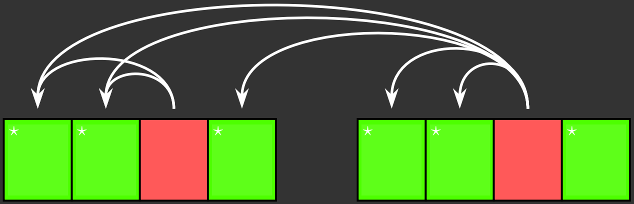
- Given $g = 0.51\ell$ oNOSF source $\mathbf{X} = (\mathbf{X}_1, \ldots, \mathbf{X}_\ell)$
- Let $\mathbf{A} = (\mathbf{X}_1, \ldots, \mathbf{X}_{\ell/2})$ and $\mathbf{B} = (\mathbf{X}_{\ell/2+1}, \ldots, \mathbf{X}_\ell)$



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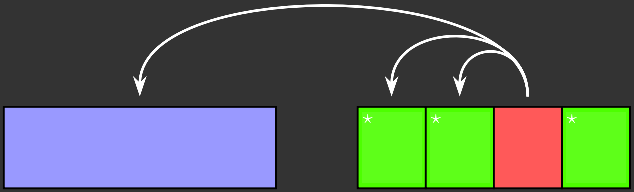
- $|\mathbf{A}| = 0.5\ell n$, $H_\infty(\mathbf{A}) \geq 0.01\ell n$



Condensing from oNOSF Sources

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Condensing from $g = 0.51\ell$, $n = 2^{\Omega(\ell)}$ oNOSF source

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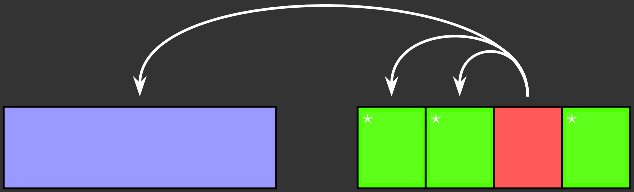
- **B** has $\geq 0.01\ell$ **good players**, and is oNOSF source



Condensing from oNOSF Sources

Condensing from entropic and $g = 1$ oNOSF source

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Condensing from $g = 0.51\ell$, $n = 2^{\Omega(\ell)}$ oNOSF source

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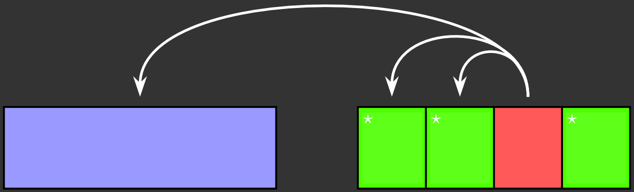
- **B** has $\geq 0.01\ell$ **good players**, and is oNOSF source
- **Cond(A, B)** from above condenses



Condensing from oNOSF Sources

Can Condense from entropic and $g = 1$ oNOSF source

A: entropic, **B**: $(\boldsymbol{g = 1, t, n \geq 2^{\Omega(t)}})$ oNOSF

Condensing from $g = 0.51\ell, \boldsymbol{n = \text{polylog}(\ell)}$ oNOSF source

Condensing from oNOSF Sources

Can Condense from entropic and $g = 1$ oNOSF source

A: entropic, **B**: $(\boldsymbol{g = 1, t, n \geq 2^{\Omega(t)}})$ oNOSF

Condensing from $g = 0.68\ell$, $\boldsymbol{n} = \textbf{poly}(\boldsymbol{\ell})$ oNOSF source

Condensing from oNOSF Sources

Can Condense from entropic and $g = 1$ oNOSF source

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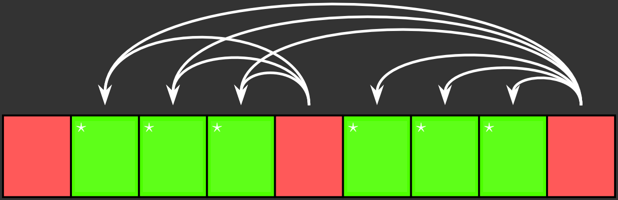
Condensing from $g = 0.68\ell$, $\boldsymbol{n = \mathrm{poly}(\ell)}$ oNOSF source

- Given $\boldsymbol{g = 0.68\ell}$ oNOSF source $\mathbf{X} = (\mathbf{X}_1, \ldots, \mathbf{X}_\ell)$



Condensing from oNOSF Sources

Can Condense from entropic and $g = 1$ oNOSF source

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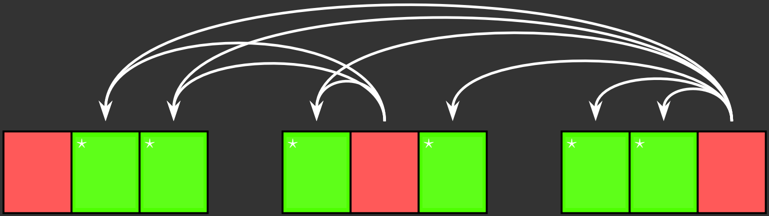
Condensing from $g = 0.68\ell$, $\boldsymbol{n = \mathrm{poly}(\ell)}$ oNOSF source

- Given $\boldsymbol{g = 0.68\ell}$ oNOSF source $\mathbf{X} = (\mathbf{X}_1, \ldots, \mathbf{X}_\ell)$
- Divide players in $\mathbf{X}$ into three equal parts - **S, A, B**



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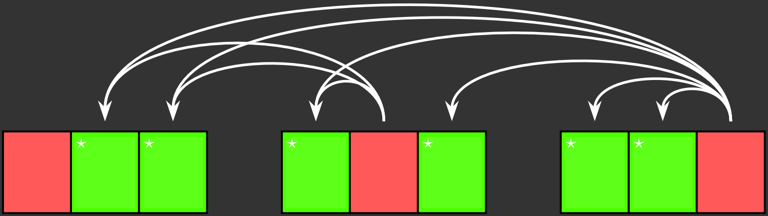

Condensing from oNOSF Sources

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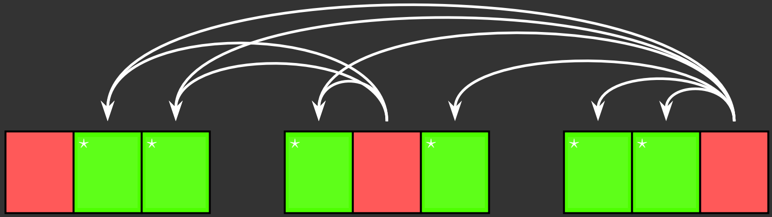
- Committee selection Russell-Zuckerman'97, Feige'98 over **S** to select $\boldsymbol{t = \log(\ell)}$ players from **B** with ≥ 1 good player whp



Condensing from oNOSF Sources

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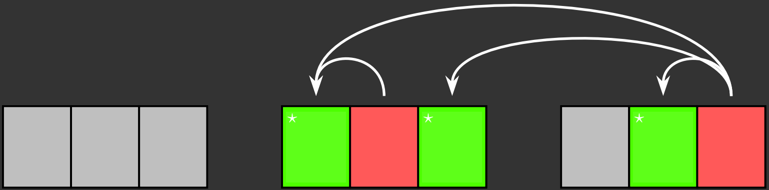

Condensing from oNOSF Sources

Can Condense from entropic and $g = 1$ oNOSF source

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Condensing from $g = 0.68\ell$, $\boldsymbol{n = \mathrm{poly}(\ell)}$ oNOSF source

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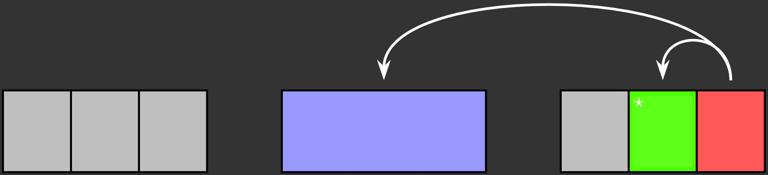
- Output $\mathbf{Cond(A, B|_{t=\log(\ell)})}$



Condensing from oNOSF Sources

Can Condense from entropic and $g = 1$ oNOSF source

A: entropic, **B**: $(g = 1, t, n = 2^{\Omega(t)})$ oNOSF

Condensing from $g = 0.68\ell, n = \text{poly}(\ell)$ oNOSF source

- Given $g = 0.68\ell$ oNOSF source $\mathbf{X} = (\mathbf{X}_1, \ldots, \mathbf{X}_\ell)$
- Divide players in **X** into three equal parts - **S, A, B**
- Each has $\geq 0.01\ell$ good players
- Committee selection Russell-Zuckerman'97, Feige'98 over **S** to select $t = \log(\ell)$ players from **B** with ≥ 1 good player whp
- Output $\text{Cond}(\mathbf{A}, \mathbf{B}|_{t=\log(\ell)})$
- Requires $n = 2^{O(t)} = \text{poly}(\ell)$

Condensing from oNOSF Sources

Can Condense from entropic and $g = 1$ oNOSF source

A: entropic, **B**: $(\boldsymbol{g = 1, t, n = 2^{\Omega(t)}})$ oNOSF

Condensing from $g = 0.68\ell$, $\boldsymbol{n = \text{poly}(\ell)}$ oNOSF source

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- Output $\mathbf{Cond(A, B|_{t=\log(\ell)})}$
- Requires $\boldsymbol{n = 2^{O(t)} = \text{poly}(\ell)}$

For $\boldsymbol{n = \text{polylog}(\ell)}$, select $\boldsymbol{t = \log(\log(\ell))}$ sized committee

Condensing from oNOSF Sources

Can Condense from entropic and $g = 1$ oNOSF source

A: entropic, **B**: $(\boldsymbol{g = 1, t, n = 2^{\Omega(t)}})$ oNOSF

Condensing from $g = 0.68\ell, \boldsymbol{n} = \textbf{poly}(\ell)$ oNOSF source

- Given $\boldsymbol{g = 0.68\ell}$ oNOSF source $\mathbf{X} = (\mathbf{X}_1, \ldots, \mathbf{X}_\ell)$
- Divide players in $\mathbf{X}$ into three equal parts - **S, A, B**
- Each has $\geq \boldsymbol{0.01\ell}$ good players
- Committee selection Russell-Zuckerman'97, Feige'98 over **S** to select $\boldsymbol{t} = \textbf{log}(\ell)$ players from **B** with ≥ 1 good player whp
- Output $\textbf{Cond}(\mathbf{A}, \mathbf{B}|_{t=\log(\ell)})$
- Requires $\boldsymbol{n = 2^{O(t)}} = \textbf{poly}(\ell)$

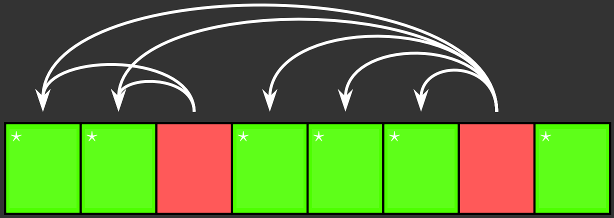
For $\boldsymbol{n} = \textbf{polylog}(\ell)$, select $\boldsymbol{t} = \textbf{log}(\textbf{log}(\ell))$ sized committee
With care, can handle $\boldsymbol{g = 0.51\ell}$

Extracting against Online Adversaries

Extracting from oNOSF sources

Extracting from oNOSF sources

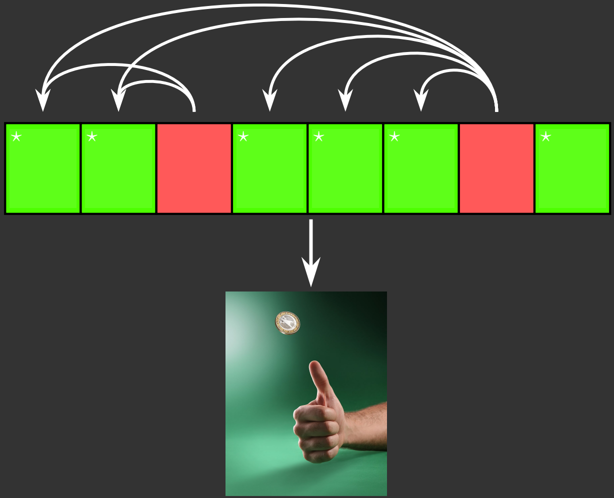
Input: (g, ℓ, n) oNOSF source



Extracting from oNOSF sources

Input: (g, ℓ, n) oNOSF source

Output: ε-close to $\mathbf{U}_1$



Extracting from oNOSF sources: Constructions

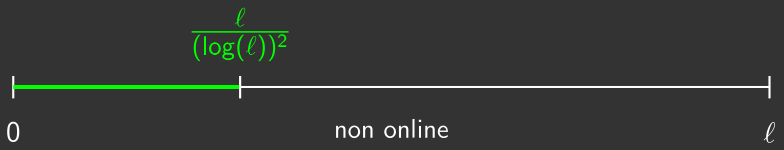
Extracting from oNOSF sources: Constructions

Extracting against non online adversaries

Extracting from oNOSF sources: Constructions

Extracting against non online adversaries

Ajtai-Linial'93 : Extract from $b = \frac{\ell}{(\log(\ell))^2}$ bad players with $n = 1$



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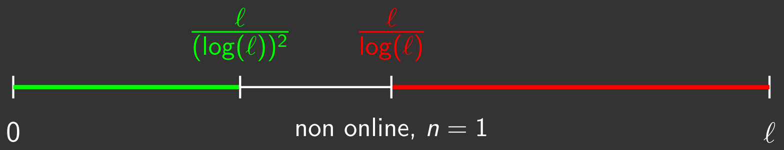

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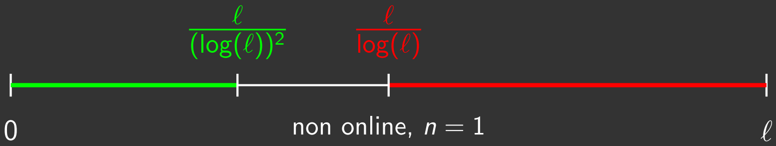
Our Results — against Online adversaries



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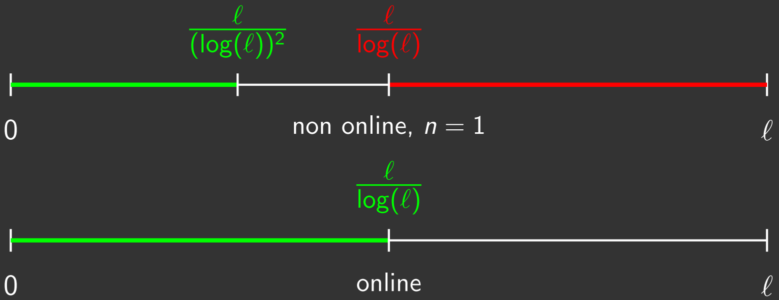
Our Results — against Online adversaries

- Extract from $\boldsymbol{b = \frac{\ell}{\log(\ell)}}$ bad players, with $\boldsymbol{n = 1}$



Extracting from oNOSF sources: Constructions

Extracting against non online adversaries

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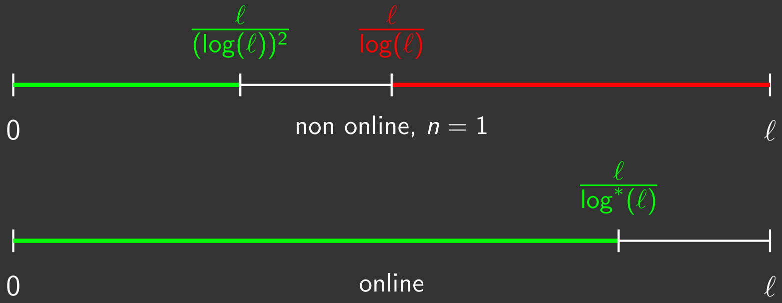
Our Results — against Online adversaries

- Extract from $b = \frac{\ell}{\log(\ell)}$ bad players, with $n = 1$
- Extract from $b = \frac{\ell}{\log^*(\ell)}$ bad players with $n = \log(\ell)$



Open Problems

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Extraction

- How many bad players can one tolerate and still extract?
- For $n = 1$, answer between $\frac{\ell}{\log(\ell)}$ and 0.01ℓ
- For $n = \log(\ell)$, answer between $\frac{\ell}{\log^*(\ell)}$ and 0.01ℓ

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Simple Explicit Optimal Condenser

- Explicitly construct seeded condenser with seed length dependence $1 \cdot \log(1/\varepsilon)$
- For our existential construction, rely on this object
- Will improve error dependence on n

Open Problems

Extraction

- How many bad players can one tolerate and still extract?
- For $\boldsymbol{n = 1}$, answer between $\frac{\ell}{\log(\ell)}$ and $\mathbf{0.01}\ell$
- For $\boldsymbol{n = \log(\ell)}$, answer between $\frac{\ell}{\log^*(\ell)}$ and $\mathbf{0.01}\ell$

Simple Explicit Optimal Condenser

- Explicitly construct seeded condenser with seed length dependence $\mathbf{1 \cdot \log(1/\varepsilon)}$
- For our existential construction, rely on this object
- Will improve error dependence on n

Condensing for $n = 1$

Show one **can condense** from $\boldsymbol{(g = 0.51\ell, \ell, n = 1)}$ oNOSF sources, or that this is **impossible**

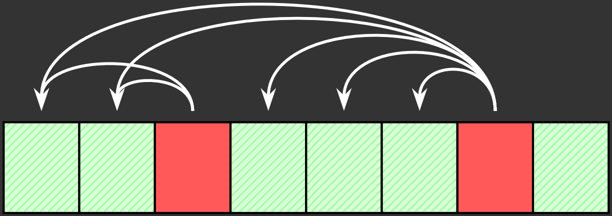
Thank You

Bonus: Converting oNOSF sources to uniform oNOSF sources

Desired transformation

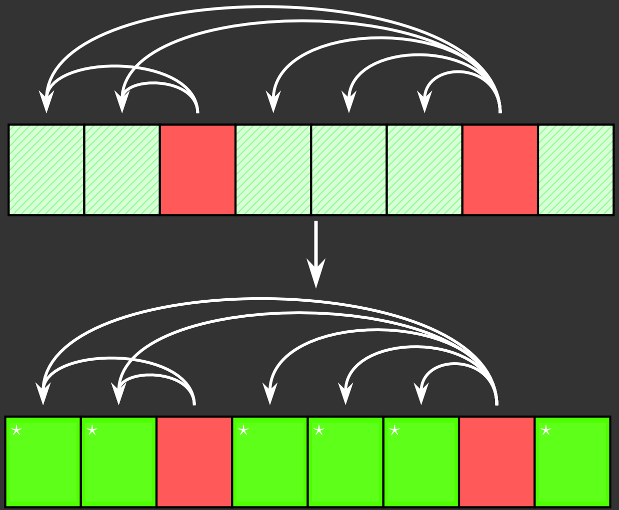
Desired transformation

Input: (g, ℓ, n, k)-oNOSF source



Desired transformation

Input: (g, ℓ, n, k)-oNOSF source
Output: ε-close to (g', ℓ', n') uniform oNOSF source



Previous Works and Our Results

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Input: (g, ℓ, n, k) oNOSF source Output: (g', ℓ', n') uniform oNOSF source

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Input: (g, ℓ, n, k) oNOSF source Output: (g', ℓ', n') uniform oNOSF source

Work	**In** k	**In** n	**Out** g	**Out** ℓ	**Out online?**

Previous Works and Our Results

Input: (g, ℓ, n, k) oNOSF source Output: (g', ℓ', n') uniform oNOSF source

Work	**In** k	**In** n	**Out** g	**Out** ℓ	**Out online?**
AORSV'20	$\geq 0.99n$	$\geq \ell^{0.01}$	$g - 1$	$\ell - 1$	No

Previous Works and Our Results

Input: (g, ℓ, n, k) oNOSF source Output: (g', ℓ', n') uniform oNOSF source

Work	**In** k	**In** n	**Out** g	**Out** ℓ	**Out online?**
AORSV'20	$\geq 0.99n$	$\geq \ell^{0.01}$	$g-1$	$\ell-1$	No
AORSV'20	$\geq 0.01n$	$\geq \ell^{0.01}$	$g-1$	100ℓ	No

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Input: (g, ℓ, n, k) oNOSF source Output: (g', ℓ', n') uniform oNOSF source

Work	**In** k	**In** n	**Out** g	**Out** ℓ	**Out online?**
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AORSV'20	$\geq 0.01n$	$\geq \ell^{0.01}$	$g-1$	100ℓ	No
CGR'24	$\geq \text{polylog}(n)$	$\geq \ell \cdot \text{polylog}(\ell)$	$g-1$	$\ell-1$	Yes

Previous Works and Our Results

Input: (g, ℓ, n, k) oNOSF source Output: (g', ℓ', n') uniform oNOSF source

Work	In k	In n	Out g	Out ℓ	Out online?
AORSV'20	$\geq 0.99n$	$\geq \ell^{0.01}$	$g-1$	$\ell-1$	No
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CGR'24	$\geq \mathrm{polylog}(n)$	$\geq \ell \cdot \mathrm{polylog}(\ell)$	$g-1$	$\ell-1$	Yes
Our Results	$\geq \mathrm{polylog}(n) + \log(\ell)$	$\geq \mathrm{polylog}(\ell)$	$g-o(\ell)$	$\ell-1$	Yes

Bonus: Extracting from oNOSF sources: Negative Results

Extracting from oNOSF sources: Negative Results

Extracting from oNOSF sources: Negative Results

AORSV'20

For $\boldsymbol{b = 0.01\ell}$ and $\boldsymbol{\varepsilon = 0.0002}$, impossible to extract from $(\ell - b, \ell, \boldsymbol{n})$-uniform oNOSF source with error $< \varepsilon$

Extracting from oNOSF sources: Negative Results

AORSV'20

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Our Results

For $\boldsymbol{b = 0.01\ell}$ and $\boldsymbol{\varepsilon = 0.0098}$ impossible to extract from $(\ell - b, \ell, \mathbf{1})$-uniform oNOSF source with error $< \varepsilon$

Extracting from oNOSF sources: Negative Techniques

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Goal: Given $f : \{0,1\}^\ell \to \{0,1\}$, corrupt $\mathbf{0.01\ell}$ variables to bias f with distance $\mathbf{0.0098}$

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For non-online setting

- Use notion of **influence** of variable for boolean function

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- Use notion of **influence** of variable for boolean function
- Prove f has **highly influential** variable Kahn-Kalai-Linial'88

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Construct tight example for online influence