Improved Parallel Repetition for GHZ-Supported Games via Spreadness

Kunal Mittal
New York University

Joint work with **Yang P. Liu** and **Shachar Lovett**

The GHZ Game [GHZ89]

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3-player game $\mathcal{G}$

P^1 P^2 P^3

Verifier: $(X, Y, Z) \sim \mathcal{S}$ **uniformly**

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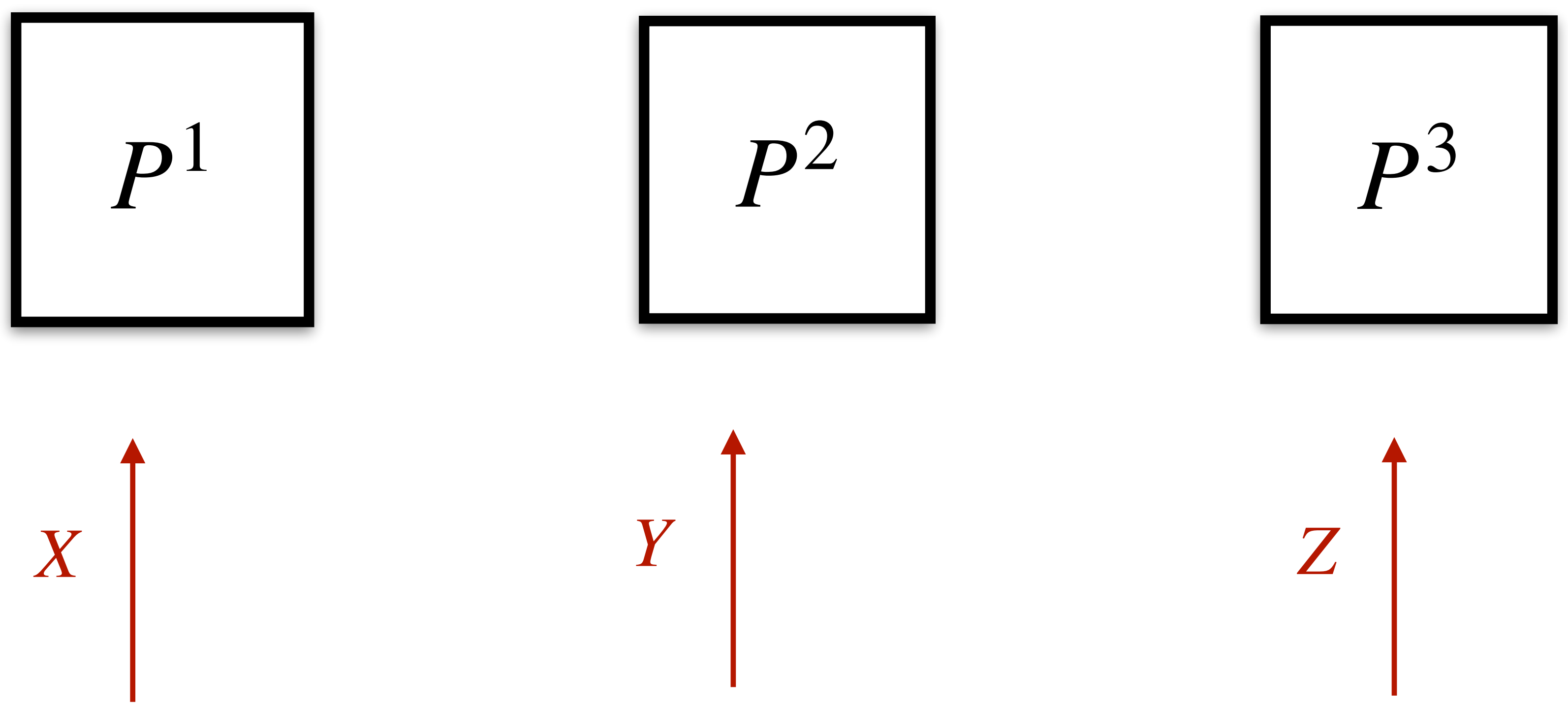

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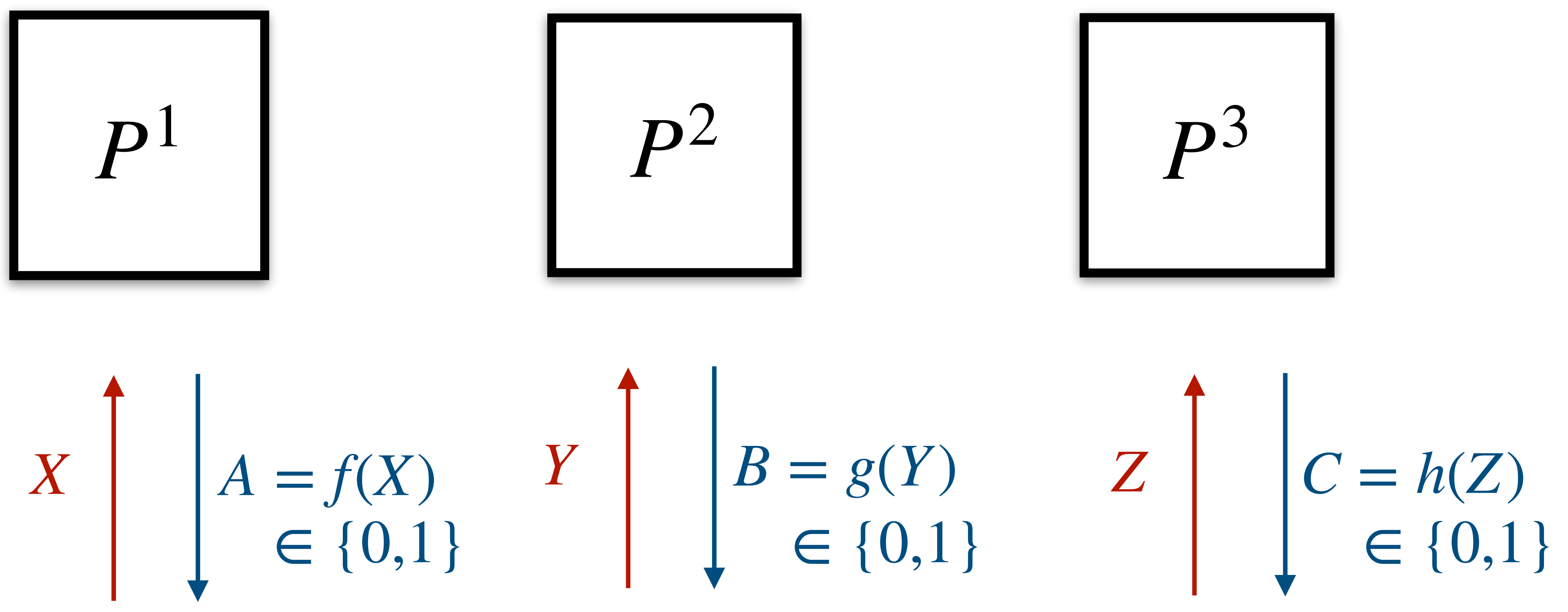

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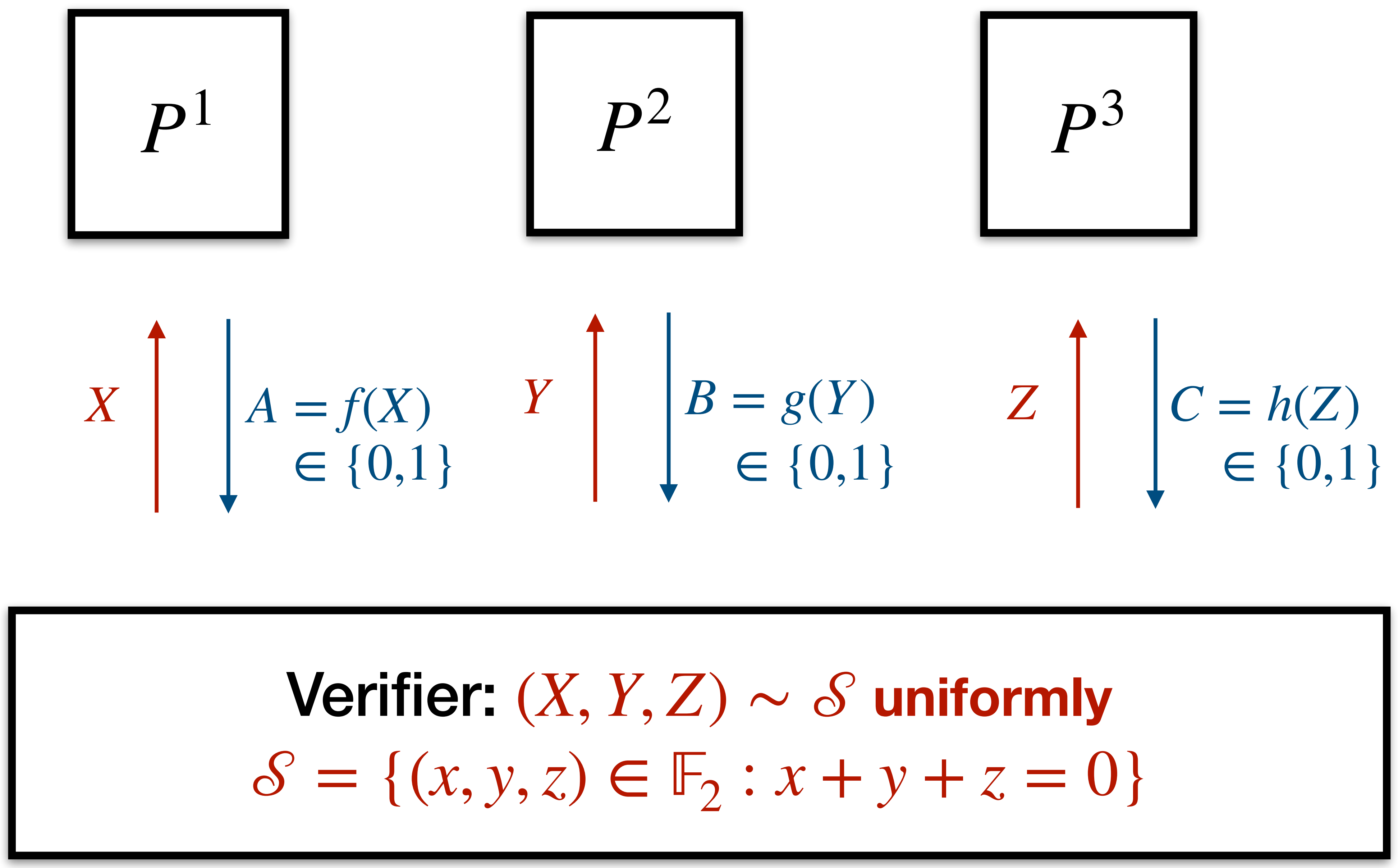

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Claim: val$(\mathscr{G}) \geq 3/4$

Proof:

$A = 1,\ B = 0,\ C = 0$ always.

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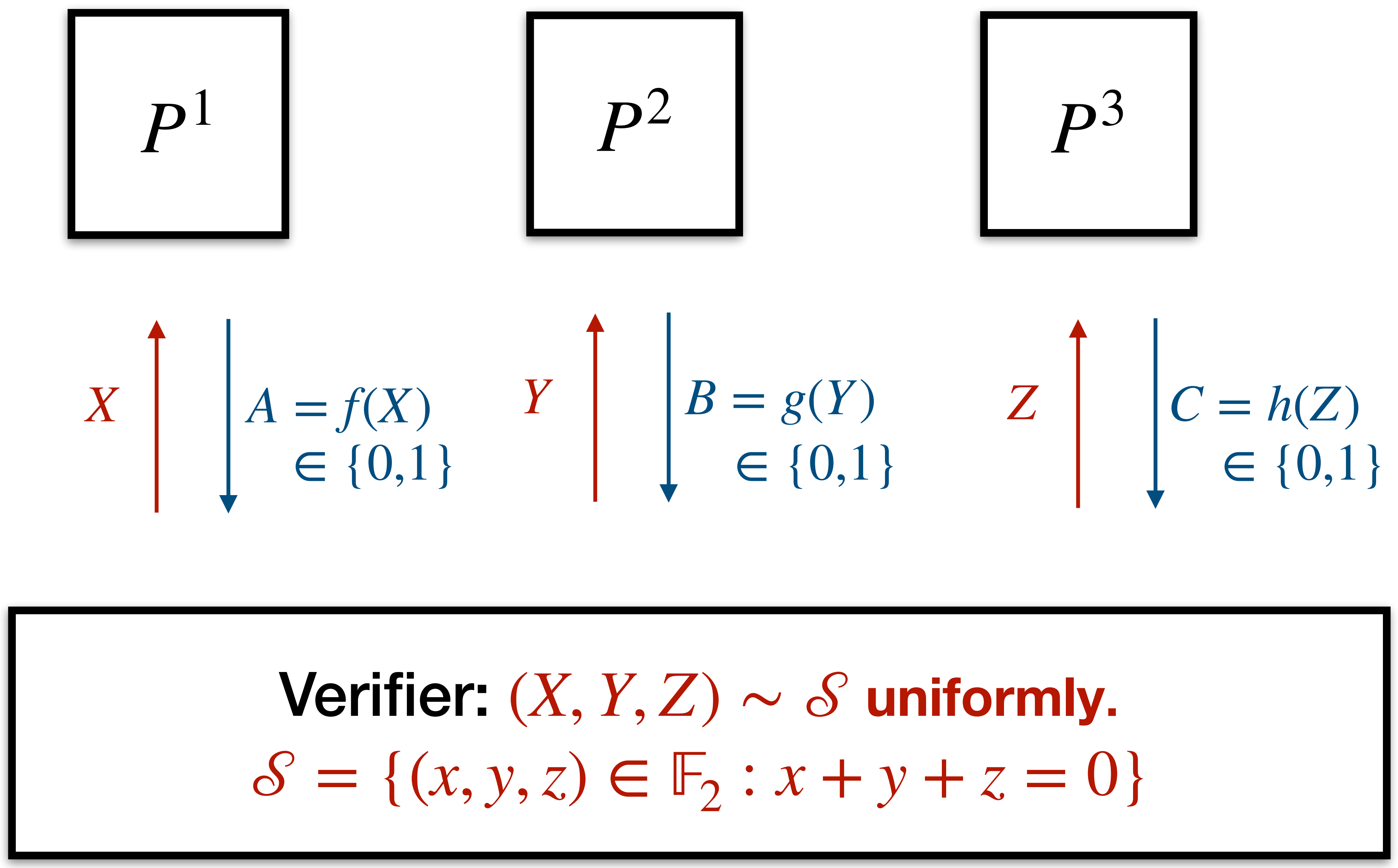

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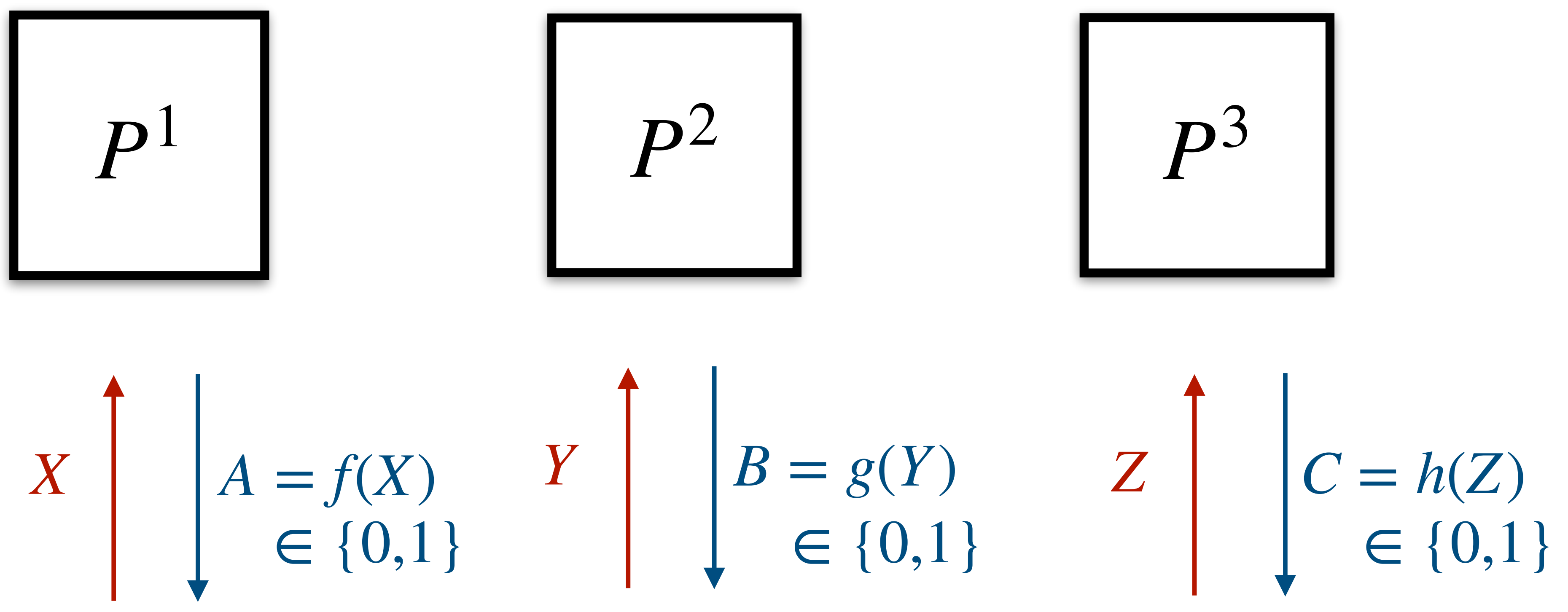

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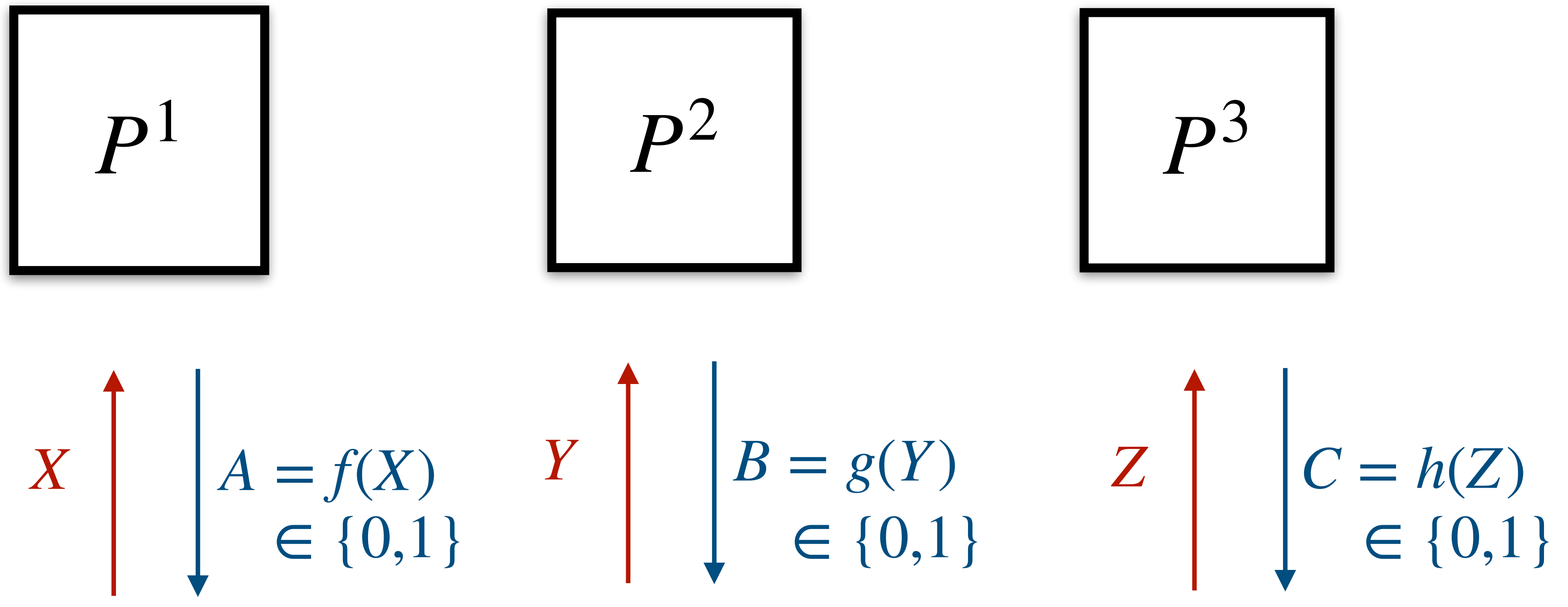

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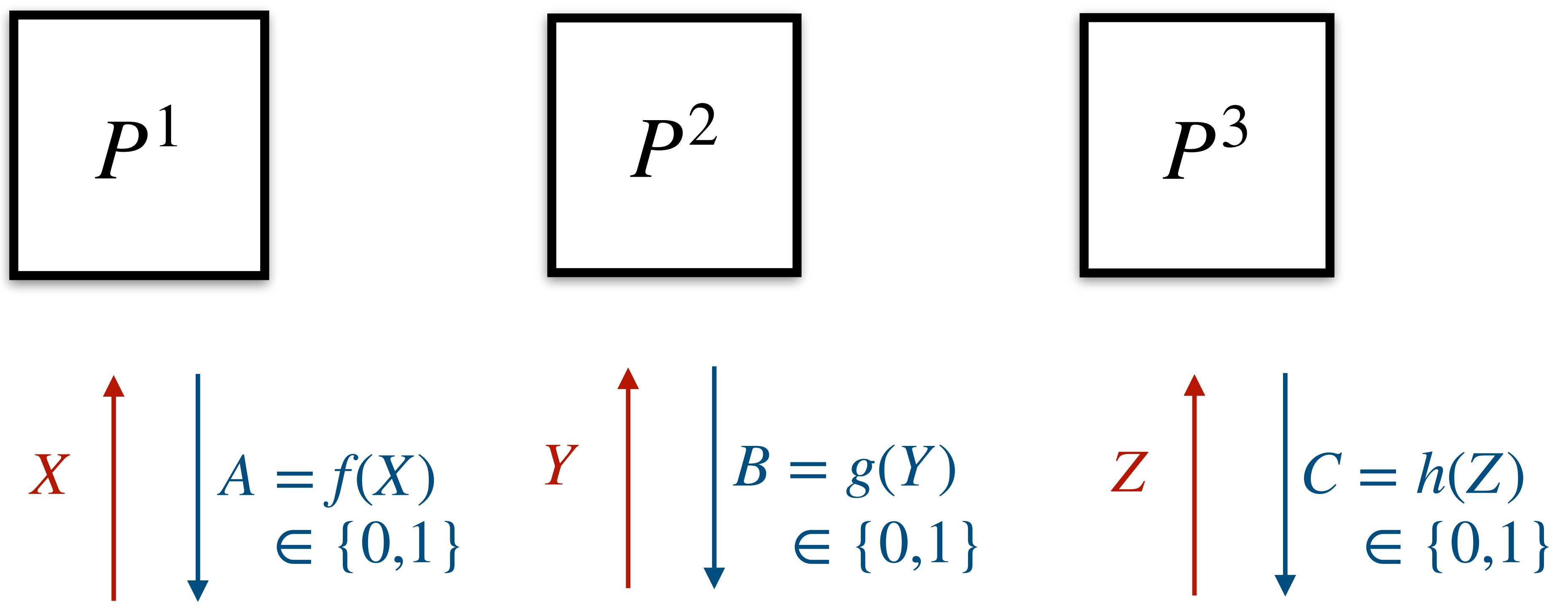

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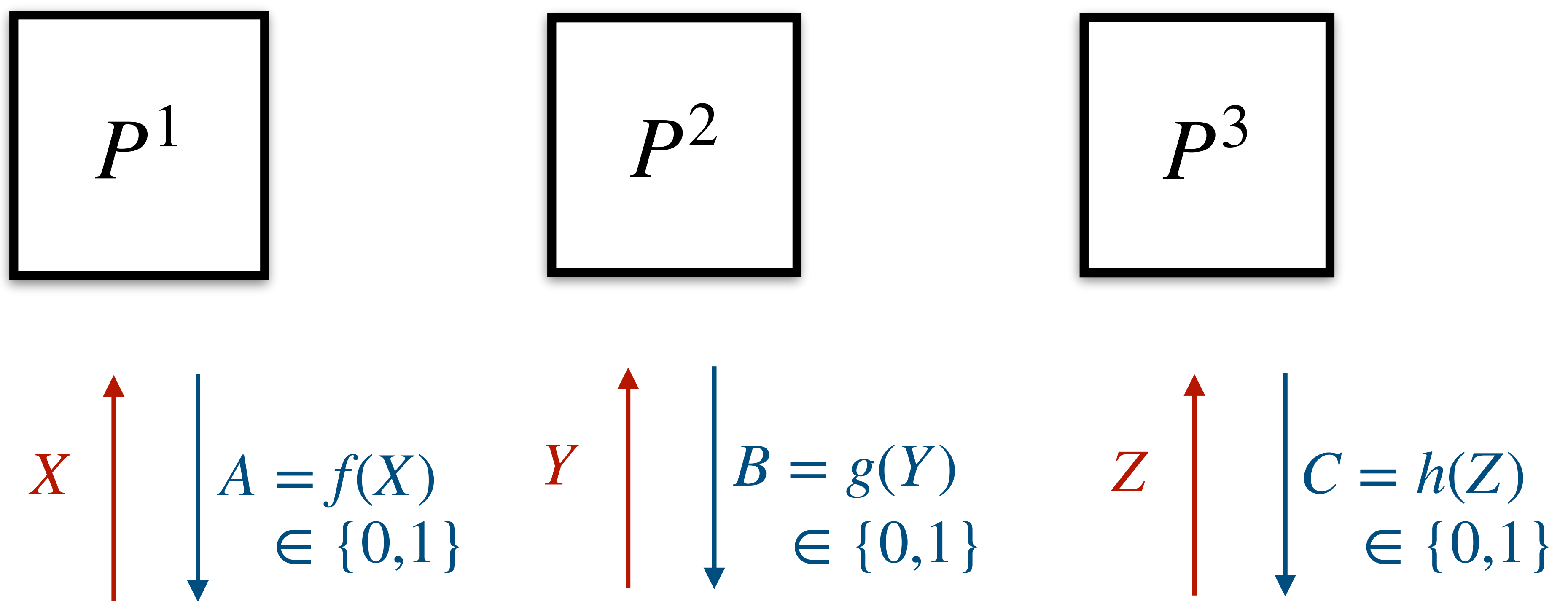

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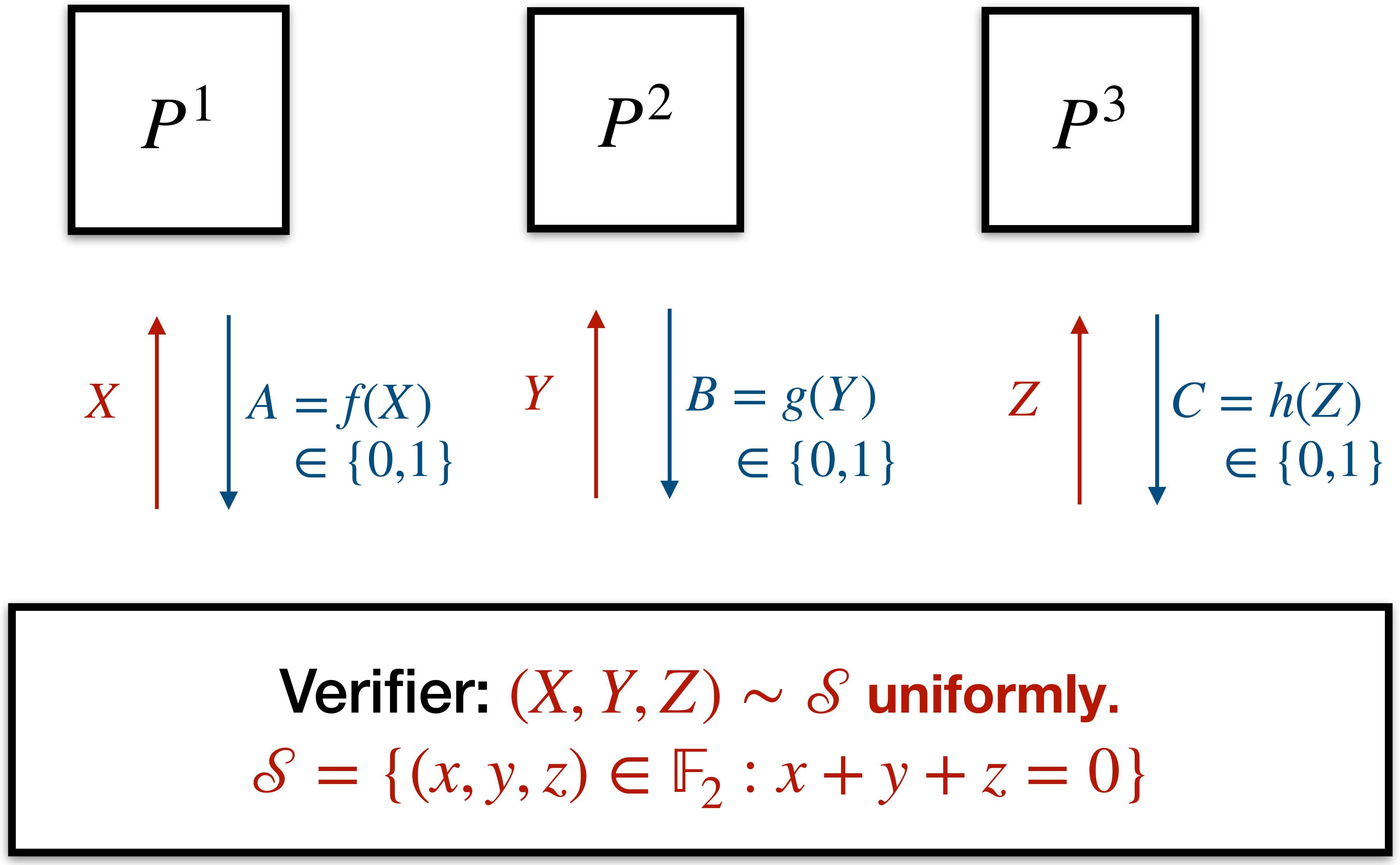

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Motivation: Quantum value is 1!

GHZ-Supported Games

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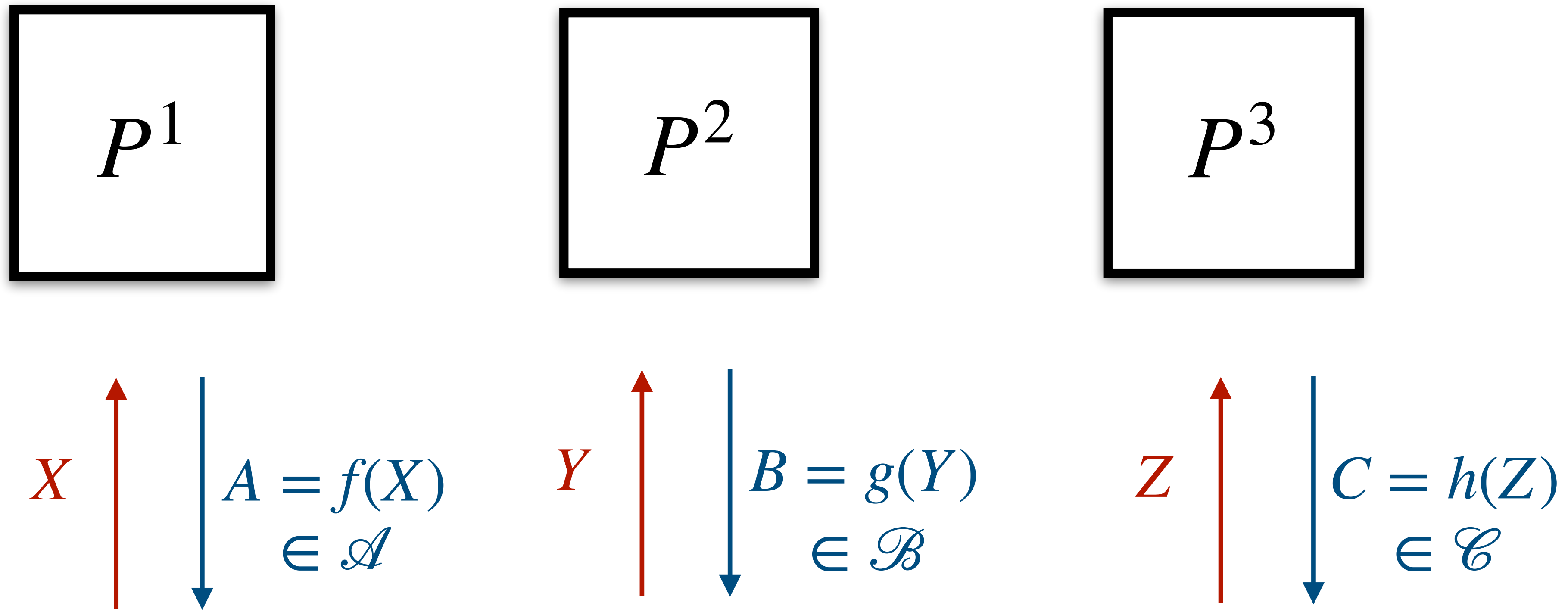

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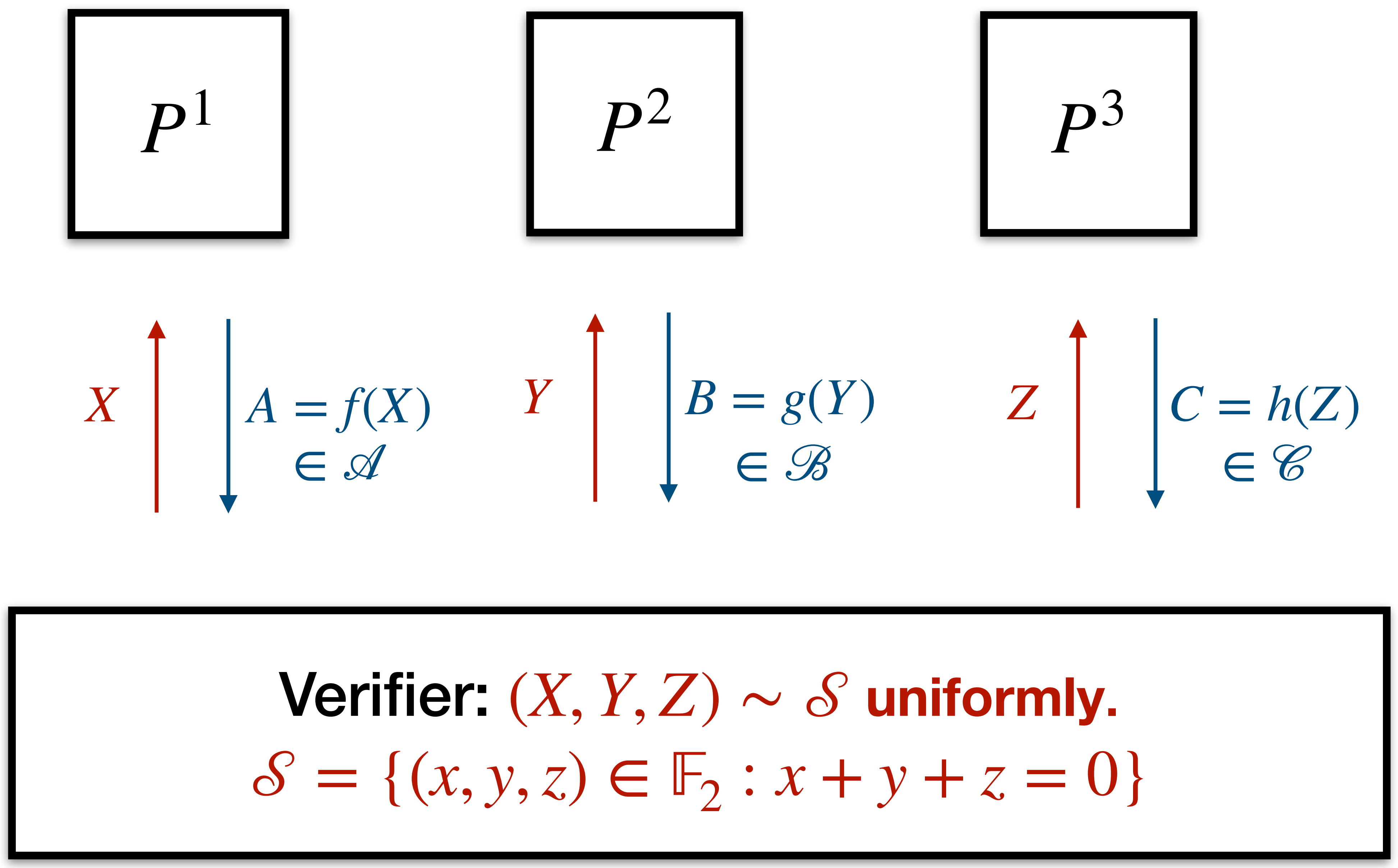

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Can similarly define general k**-player games**:

- Verifier: $(X^{(1)}, X^{(2)}, \ldots, X^{(k)}) \sim \mu$
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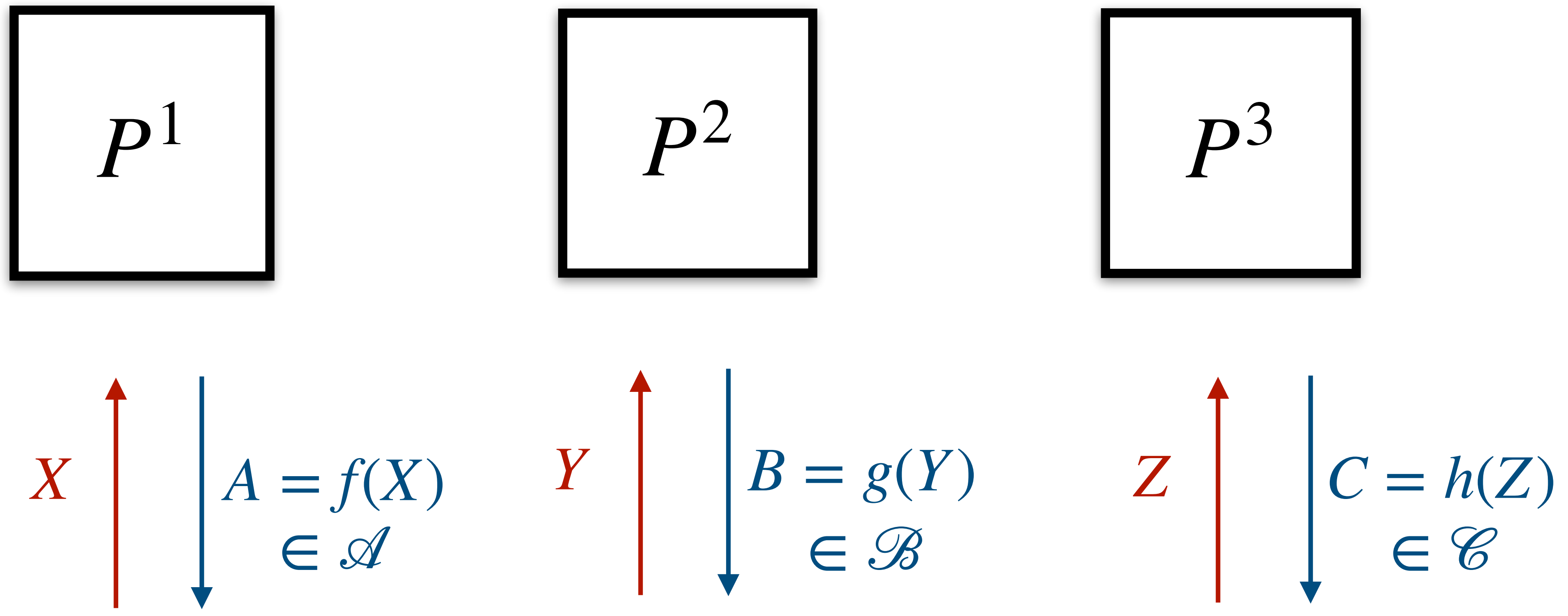

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Motivation: Interactive Proofs and PCPs

Parallel Repetition: Soundness Amplification

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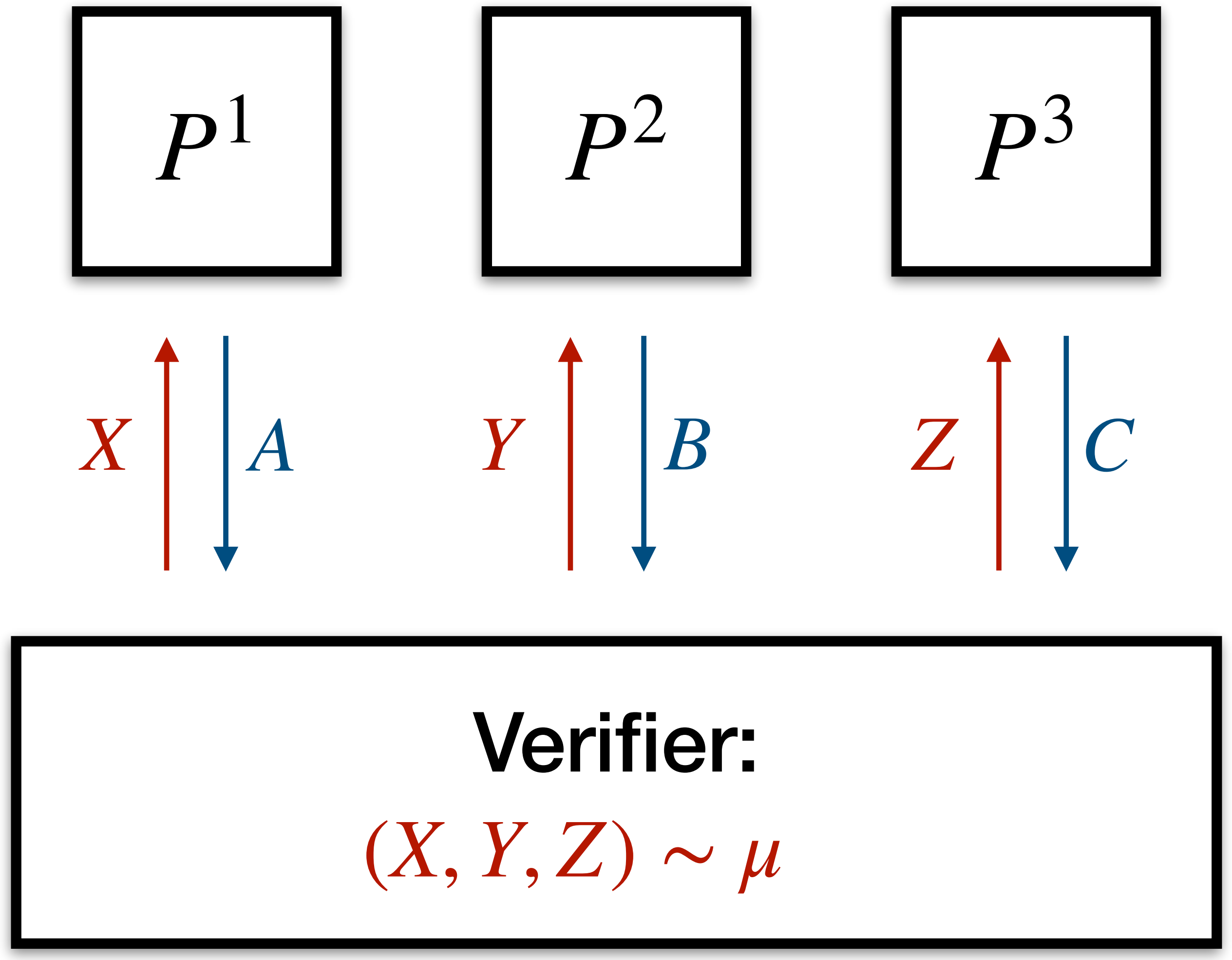

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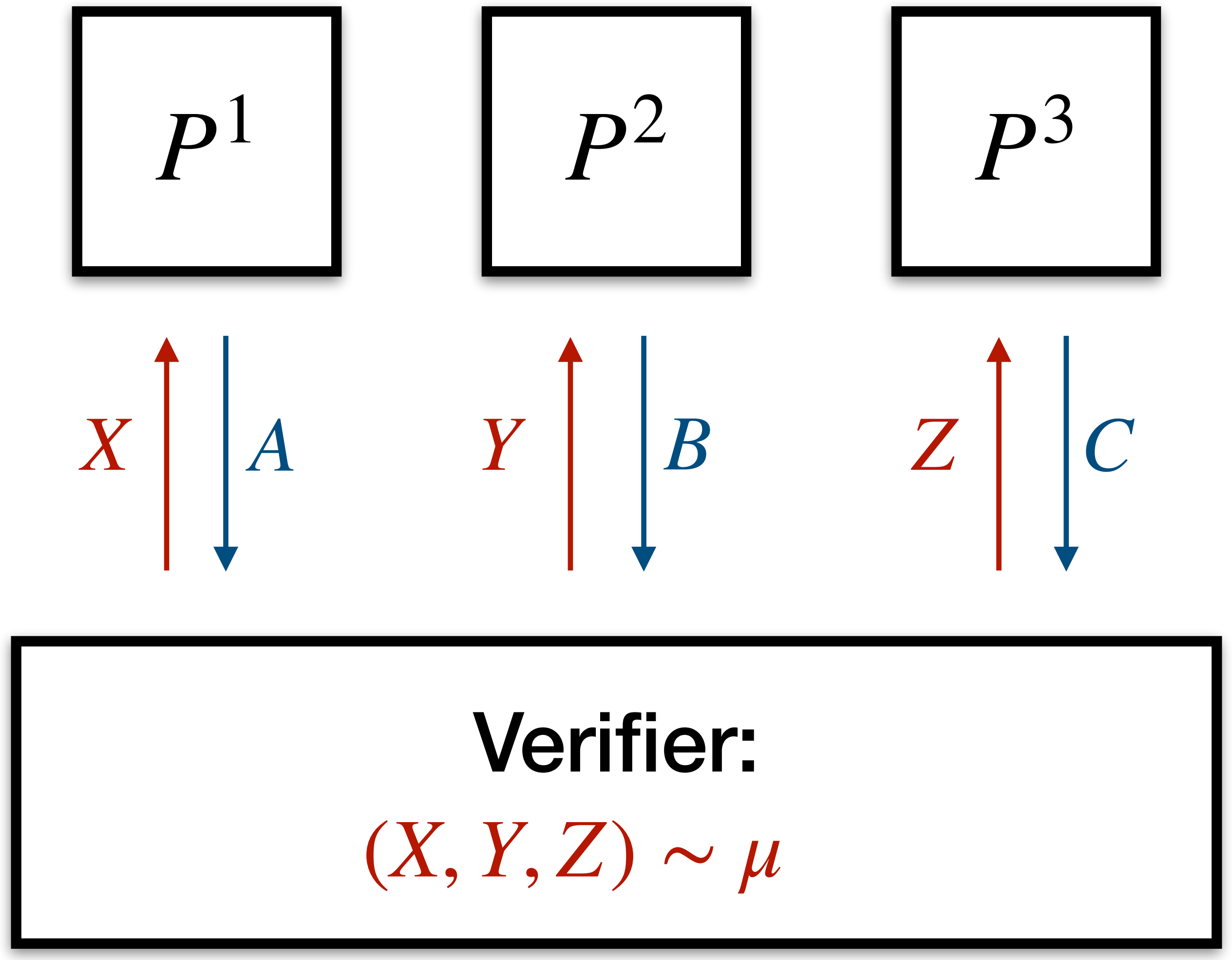

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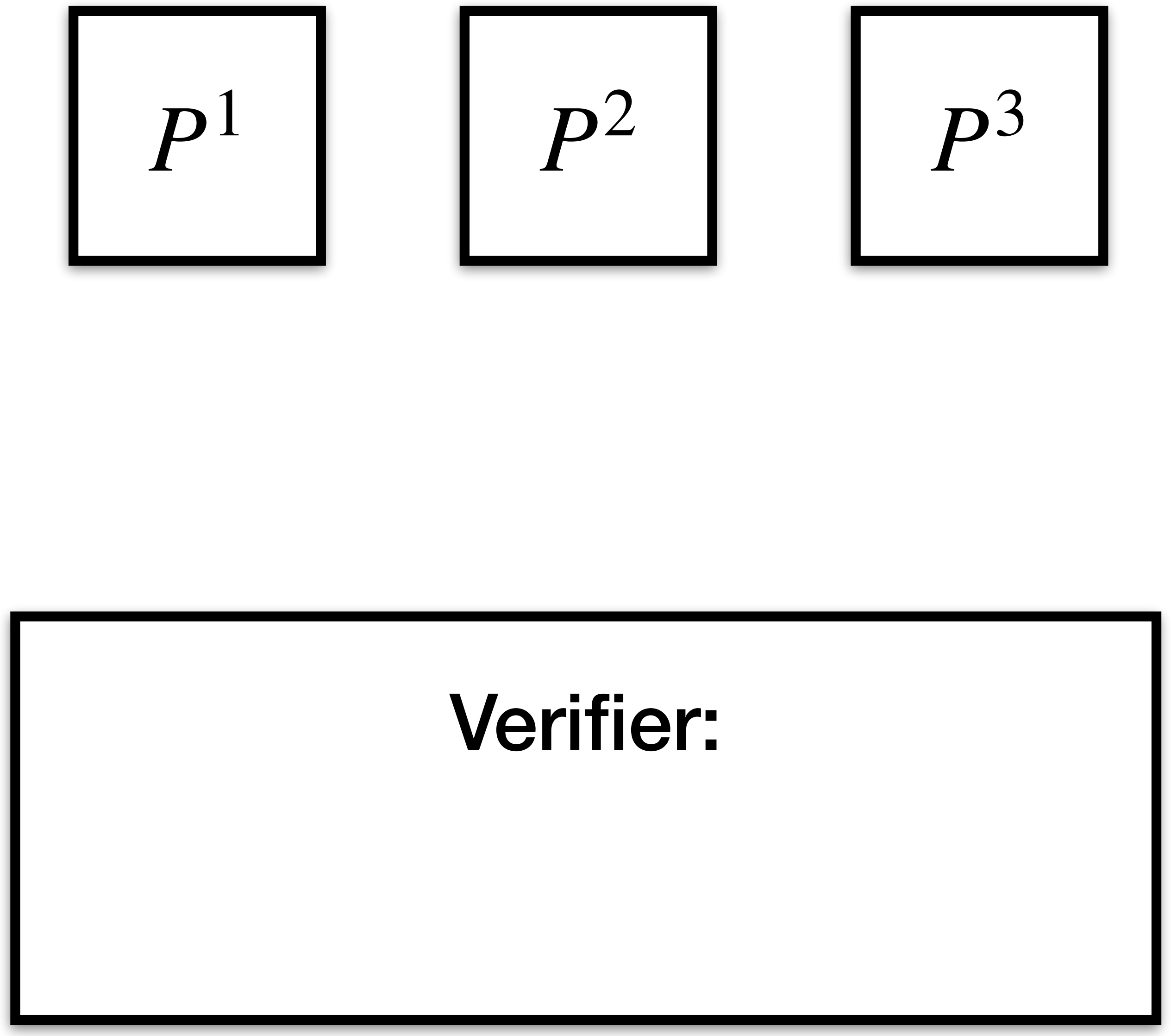

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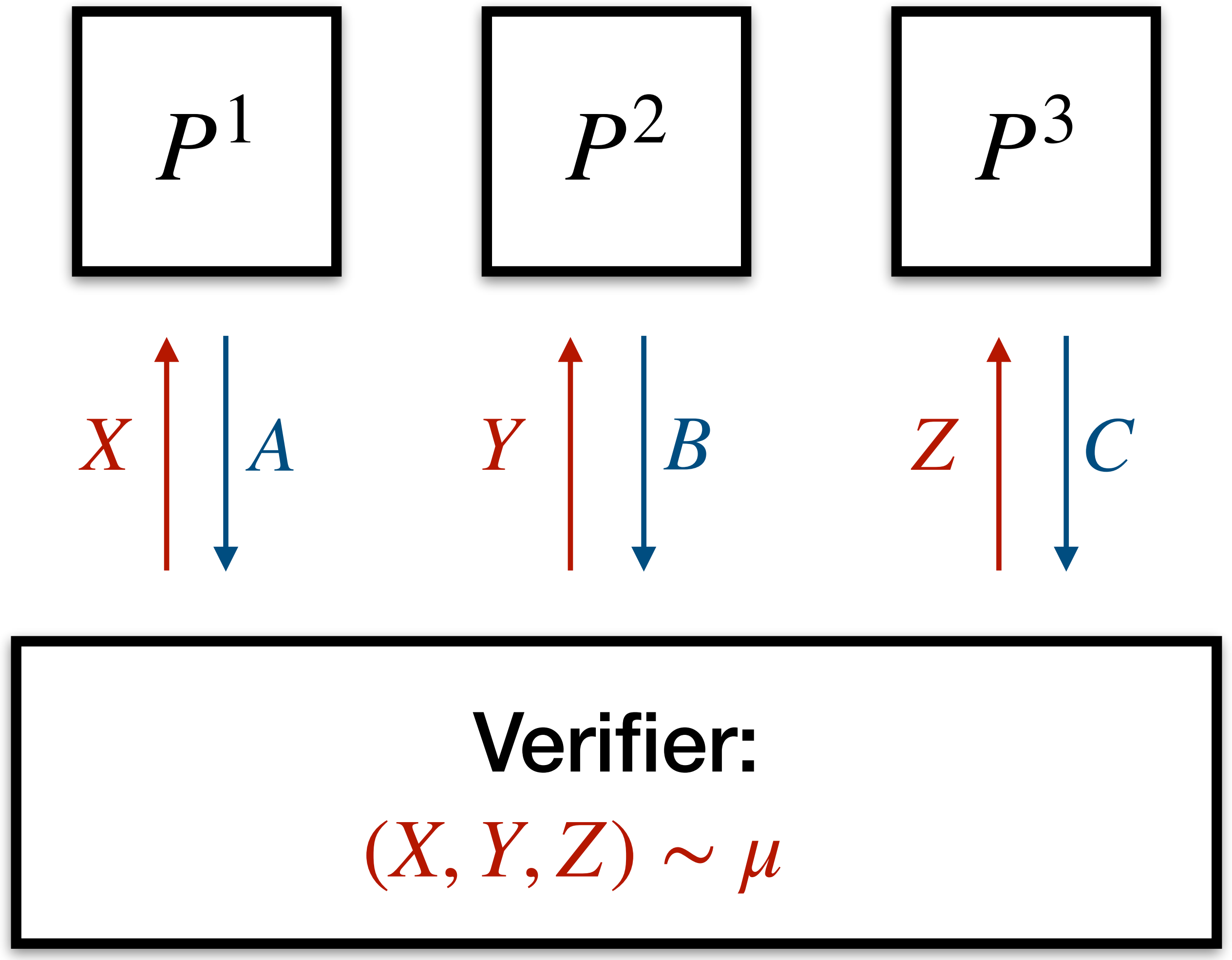

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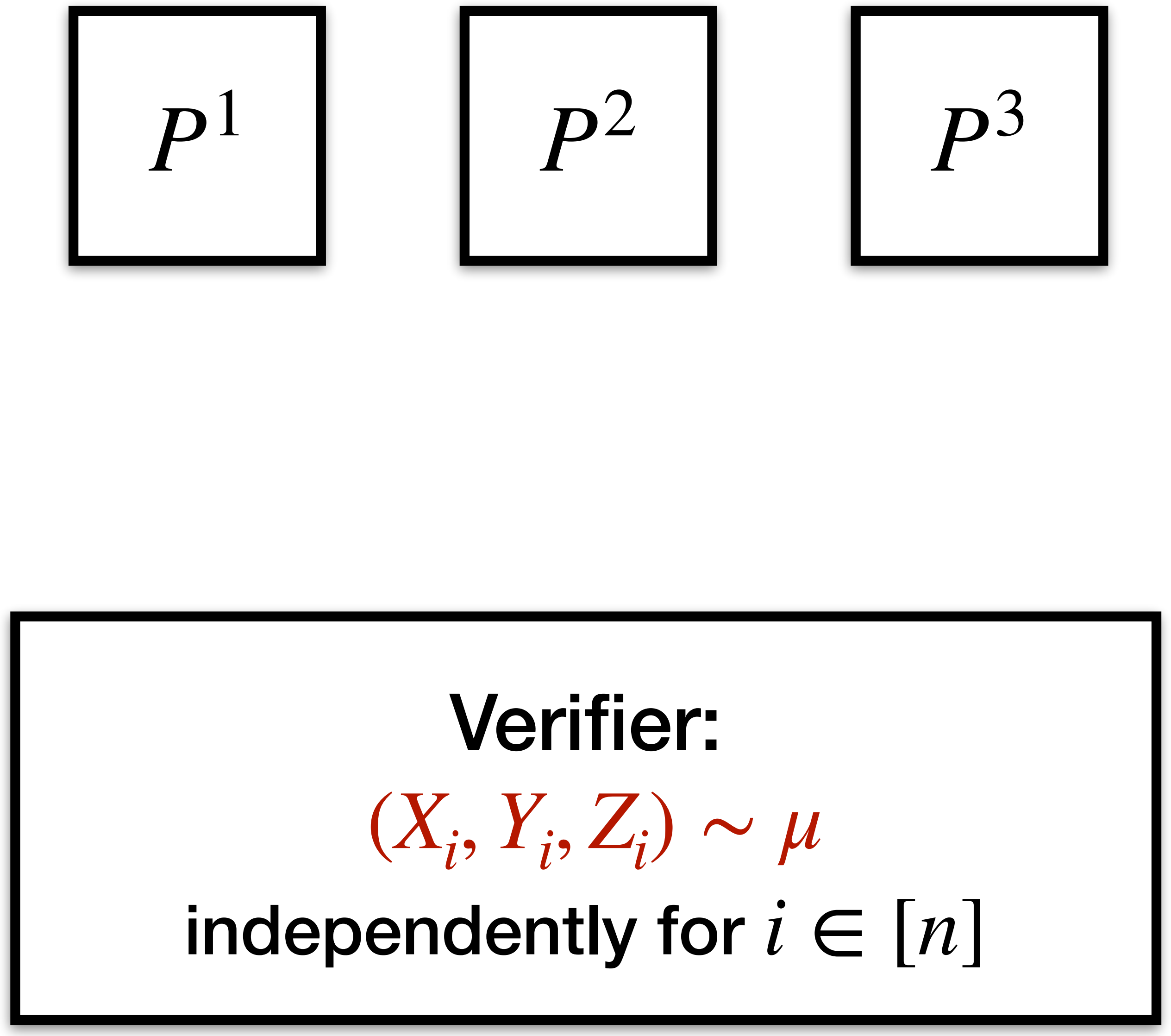

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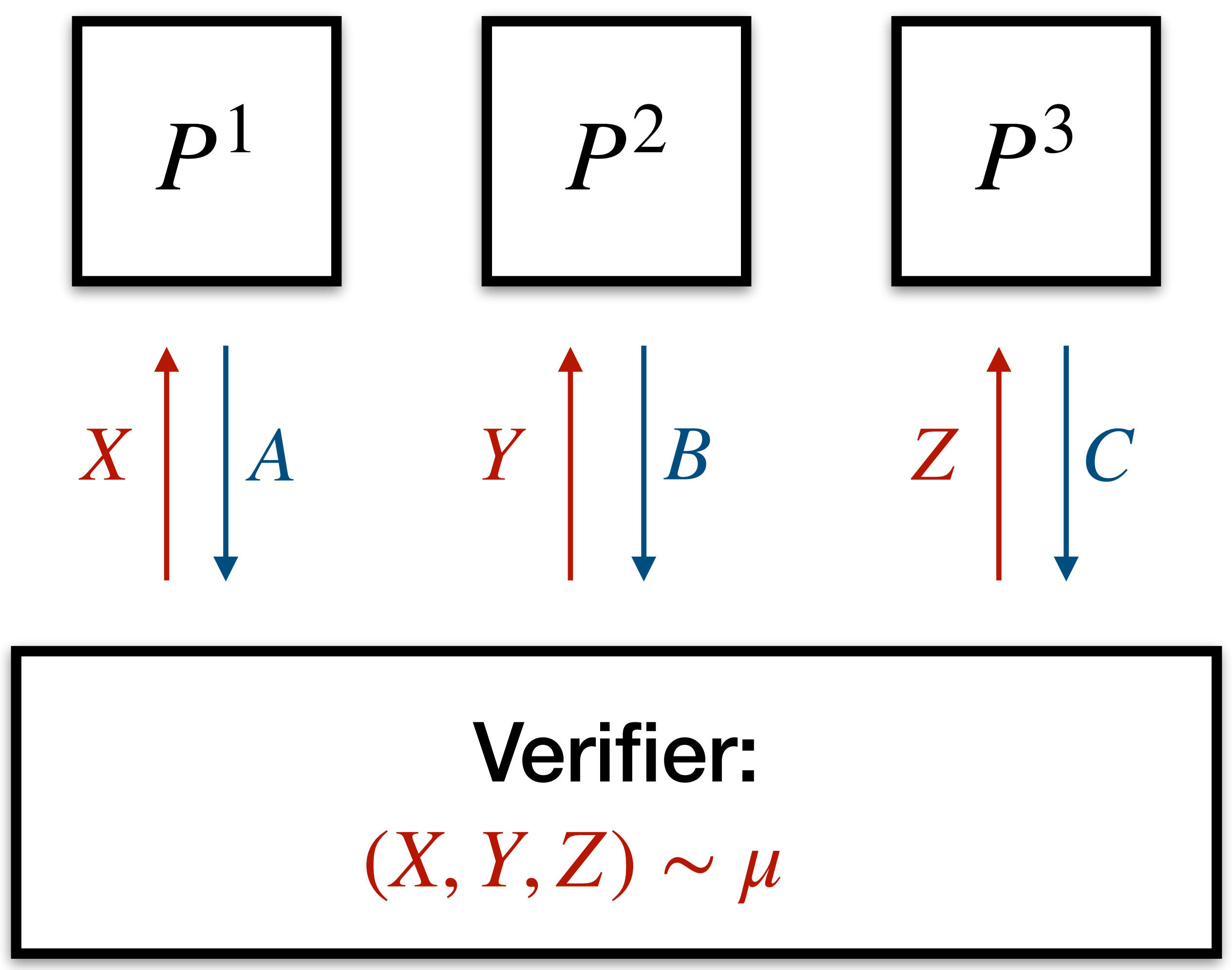

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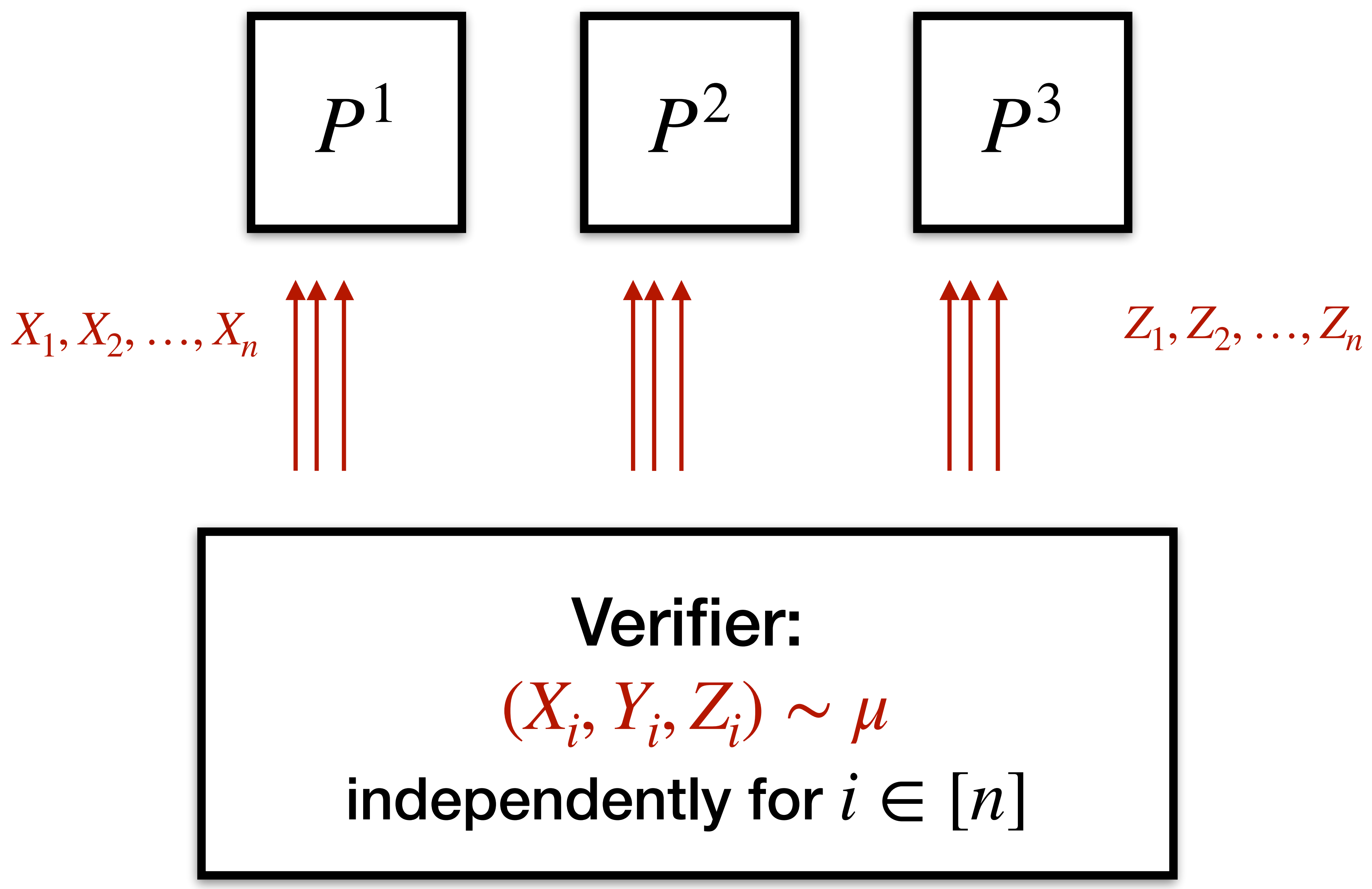

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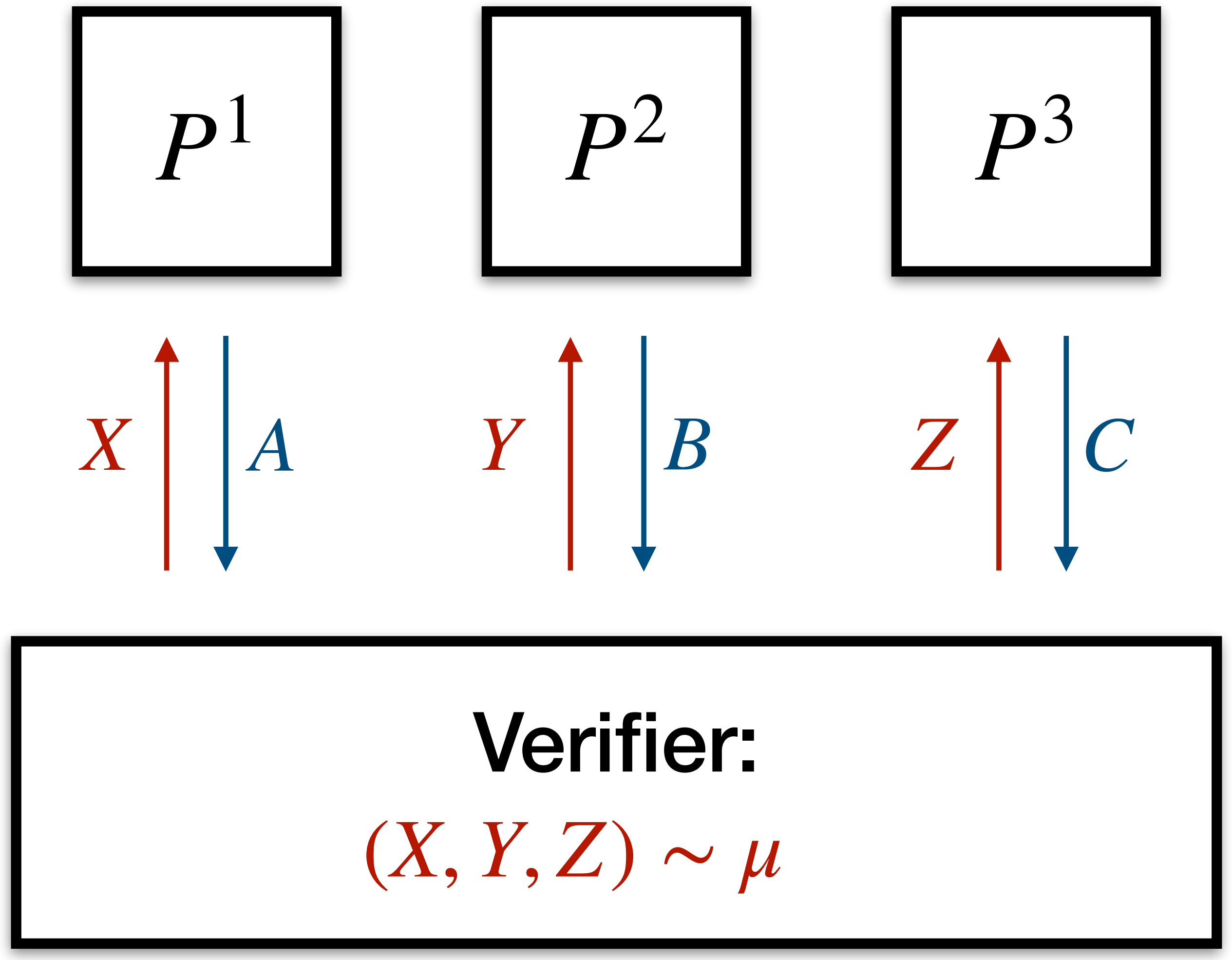

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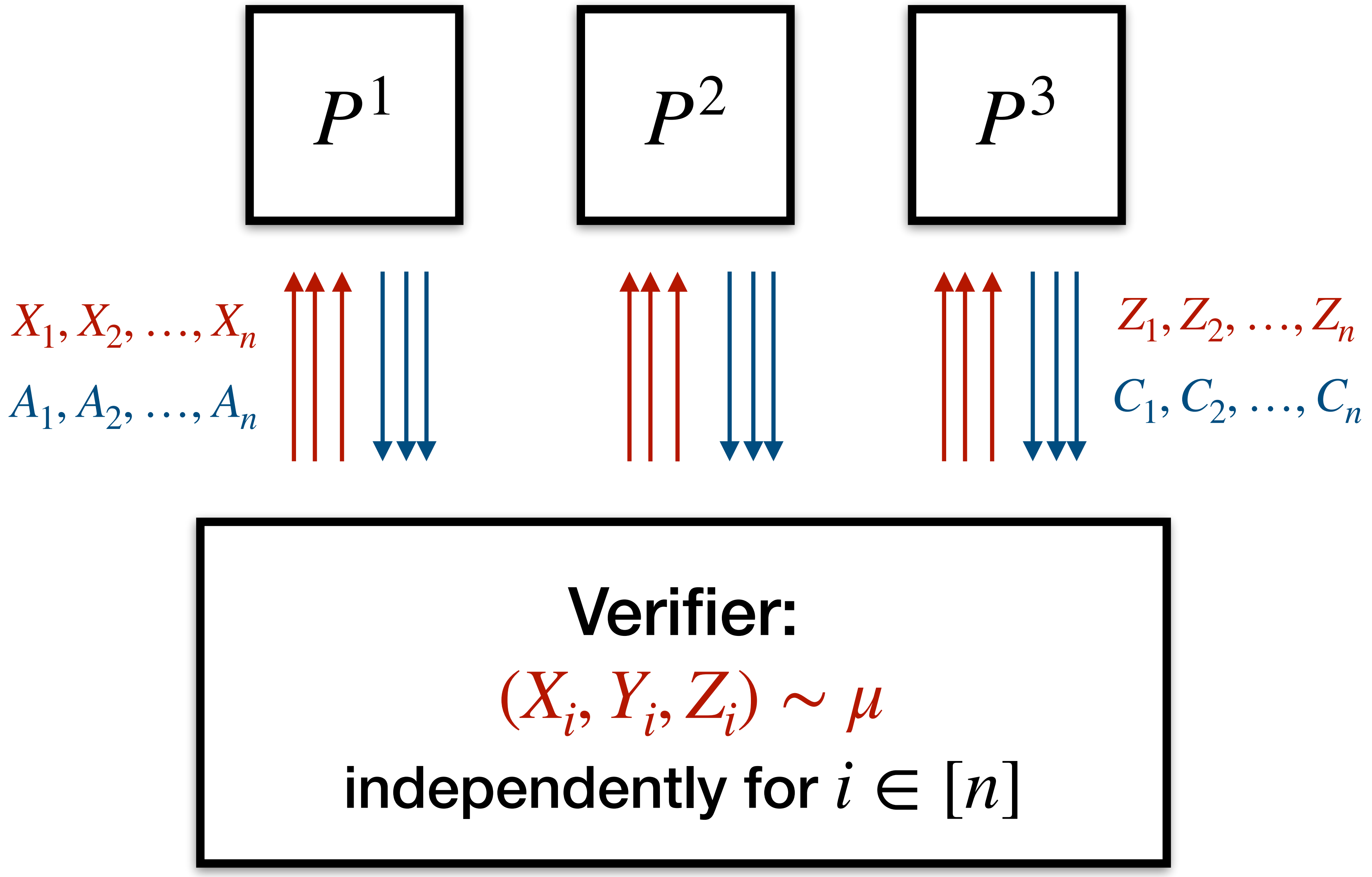

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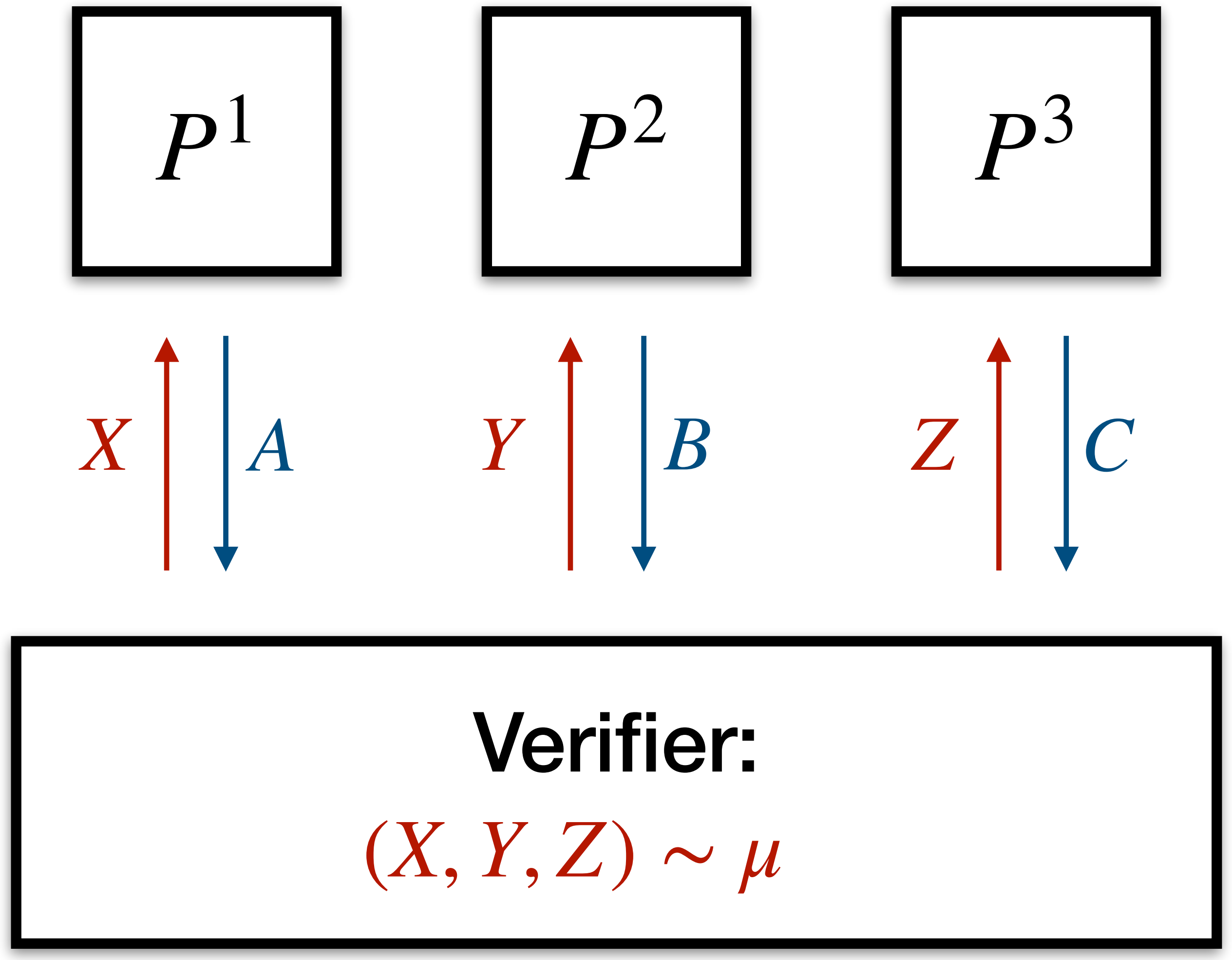

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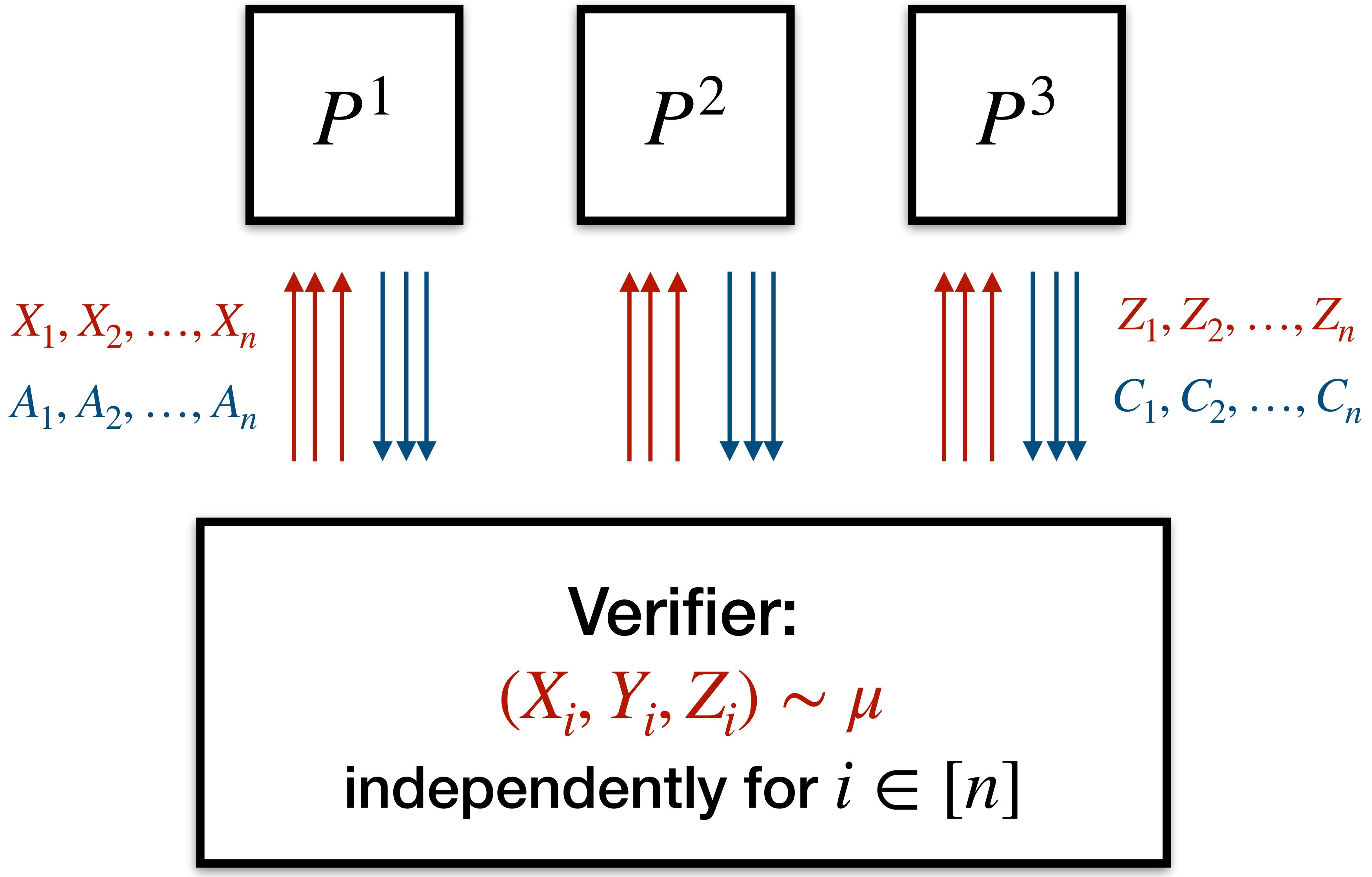

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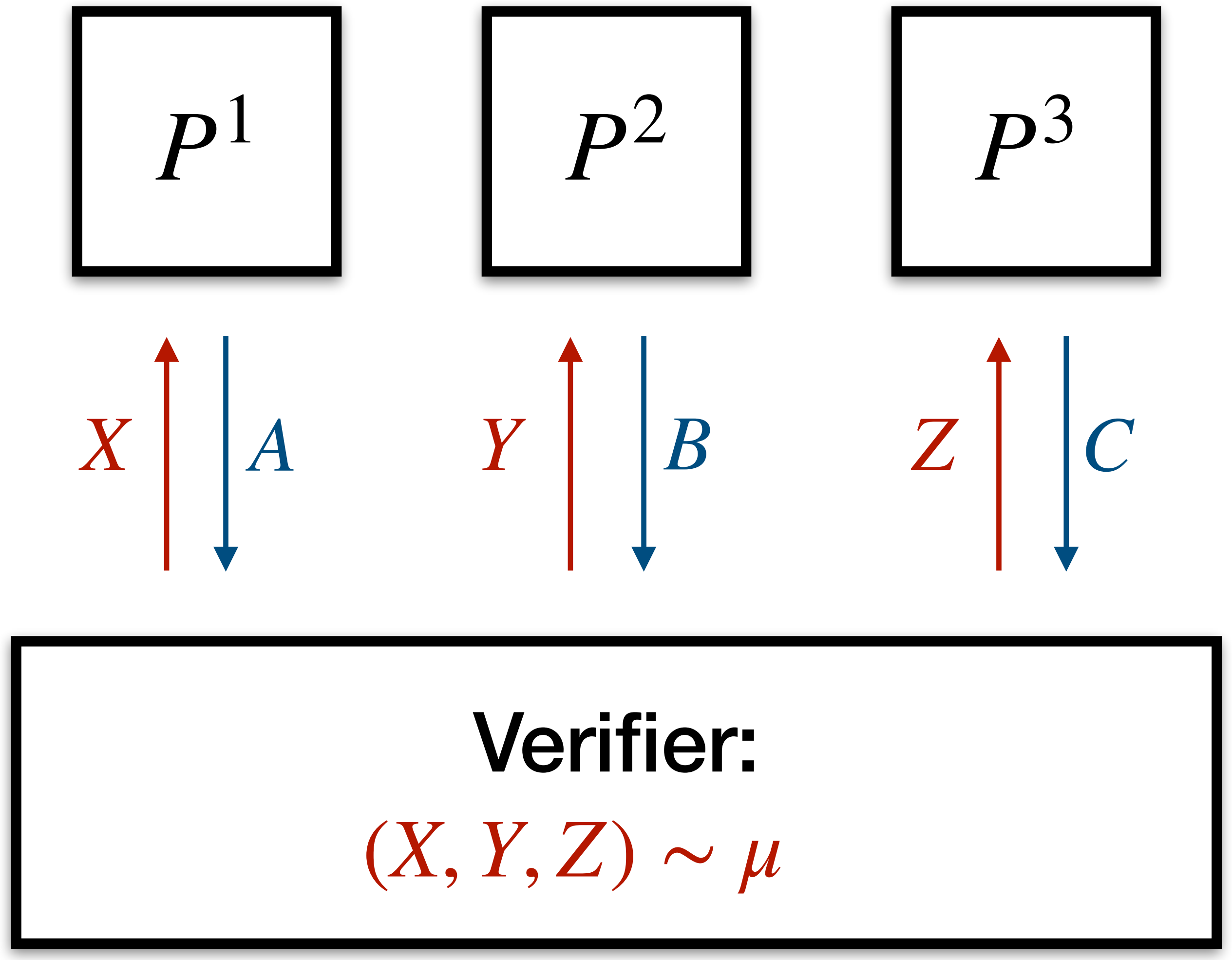

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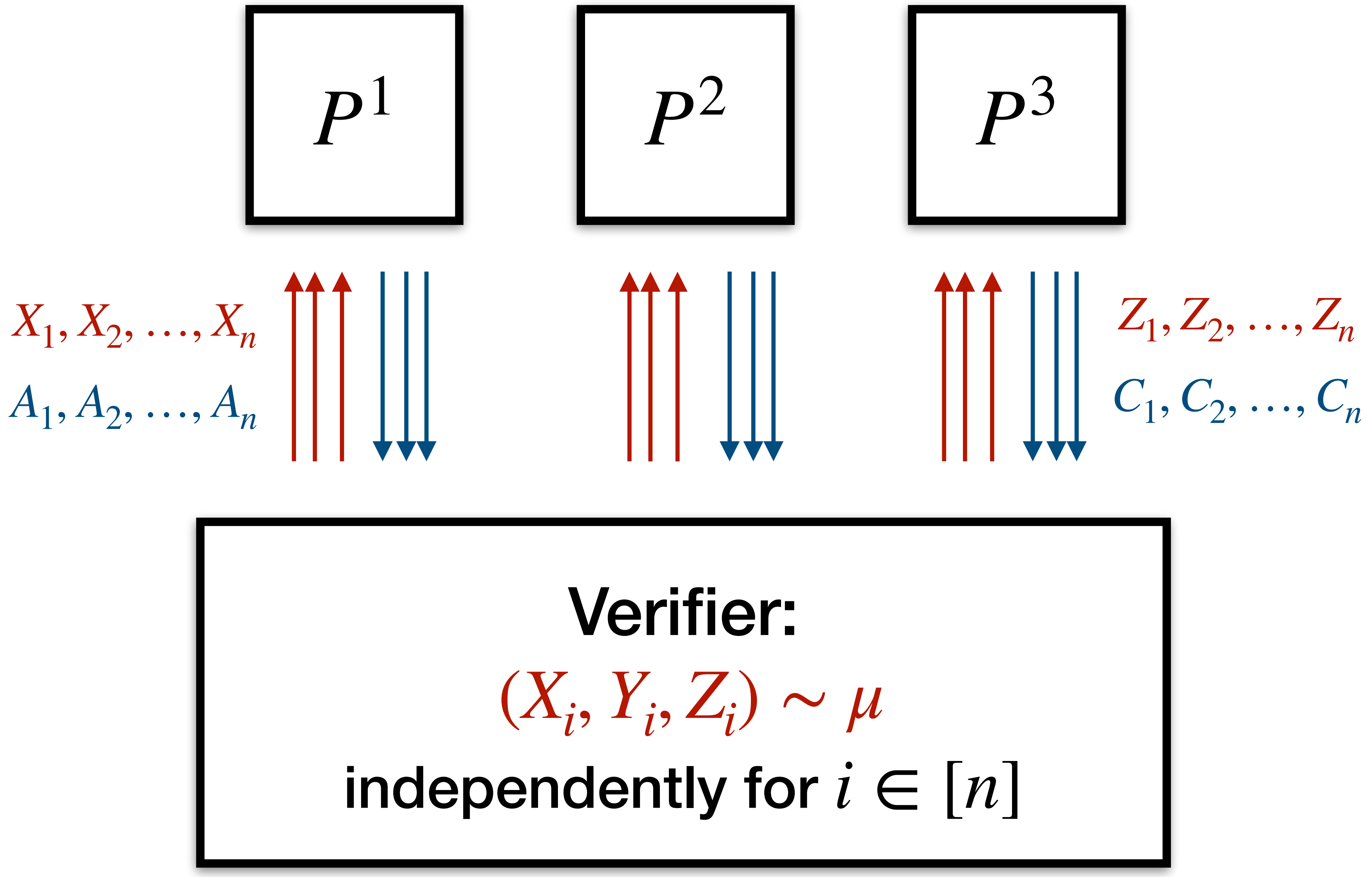

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Similarly for general **k-player games**

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Parallel Repetition

Question [BGKW88, FRS94]: What can we say about $\mathbf{val}(\mathscr{G}^{\otimes n})$?

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- [DHVY17] Exponential bounds for "connected games"
- [HR20, GHMRZ21, GHMRZ22, GMRZ22, BKM23, BBKLM25, BBKLMM26] Better decay bounds for different special classes of games.

Parallel Repetition: Applications

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2 player Games:

- PCPs and Hardness of Approximation [BGS98, Hås01, Fei96, …]
- Geometry of Foams (tiling the space) [FKO07, KORW08, AK09]
- Quantum Information [CHTW04]
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Multiplayer Games:

- [MR21] Strong parallel repetition for a class of games implies super-linear lower bounds for non-uniform Turing machines
- [FV02, HHR16, M'25] When answer sizes large, equivalent to problems in extremal combinatorics; e.g., GHZ is same as square-free sets in $\mathbb{F}_2^n \times \mathbb{F}_2^n$
- Techniques might be useful for direct sum/product theorems in NOF communication complexity

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[DHVY17] "We believe that the strong correlations present in the GHZ question distribution represent the 'hardest instance' of the multiplayer parallel repetition problem."

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- [HR20, GHMRZ21] $\quad \text{val}(\mathcal{G}^{\otimes n}) \leq n^{-c}$ $\quad$ Fourier analysis: for all GHZ supported games
- [BKM23] $\quad \text{val}(\mathcal{G}^{\otimes n}) \leq 2^{-\Omega(n)}$ $\quad$ Additive combinatorics: only for the specific GHZ game

($a \oplus b \oplus c = x \vee y \vee z$)

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A High Level Proof Outline

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- **Goal:** Show $\Pr[\mathrm{Win}] \leq \exp(-n^c)$.

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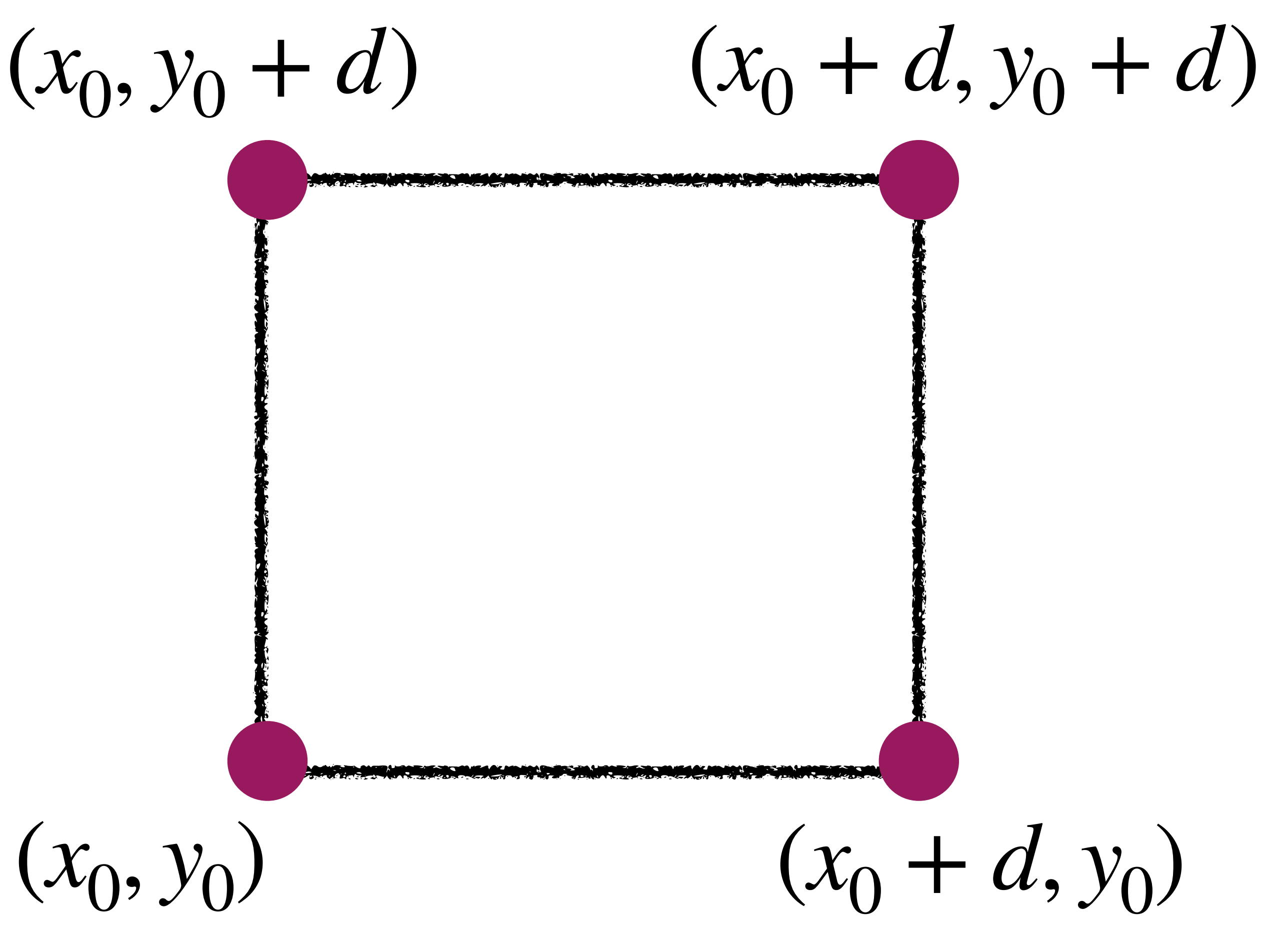

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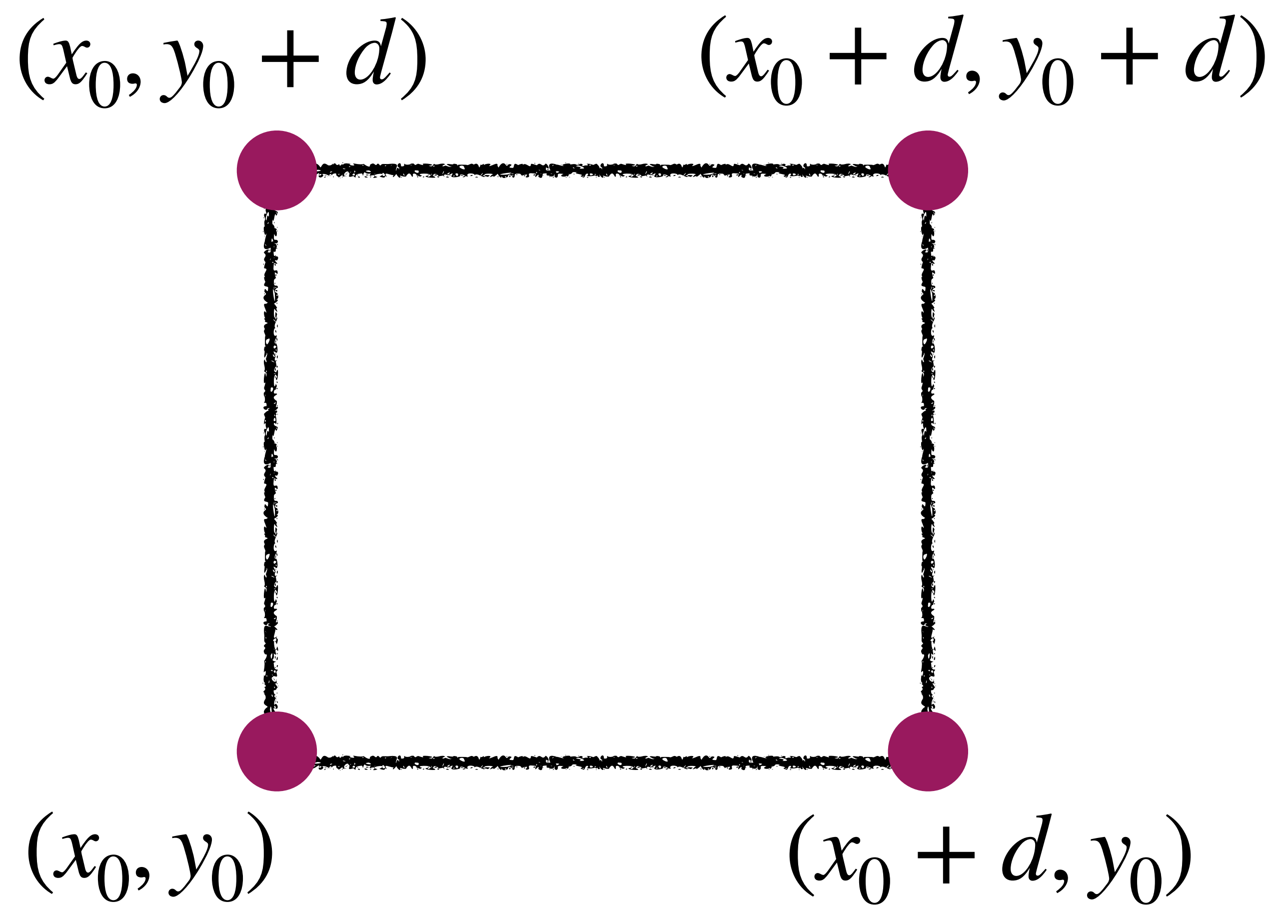

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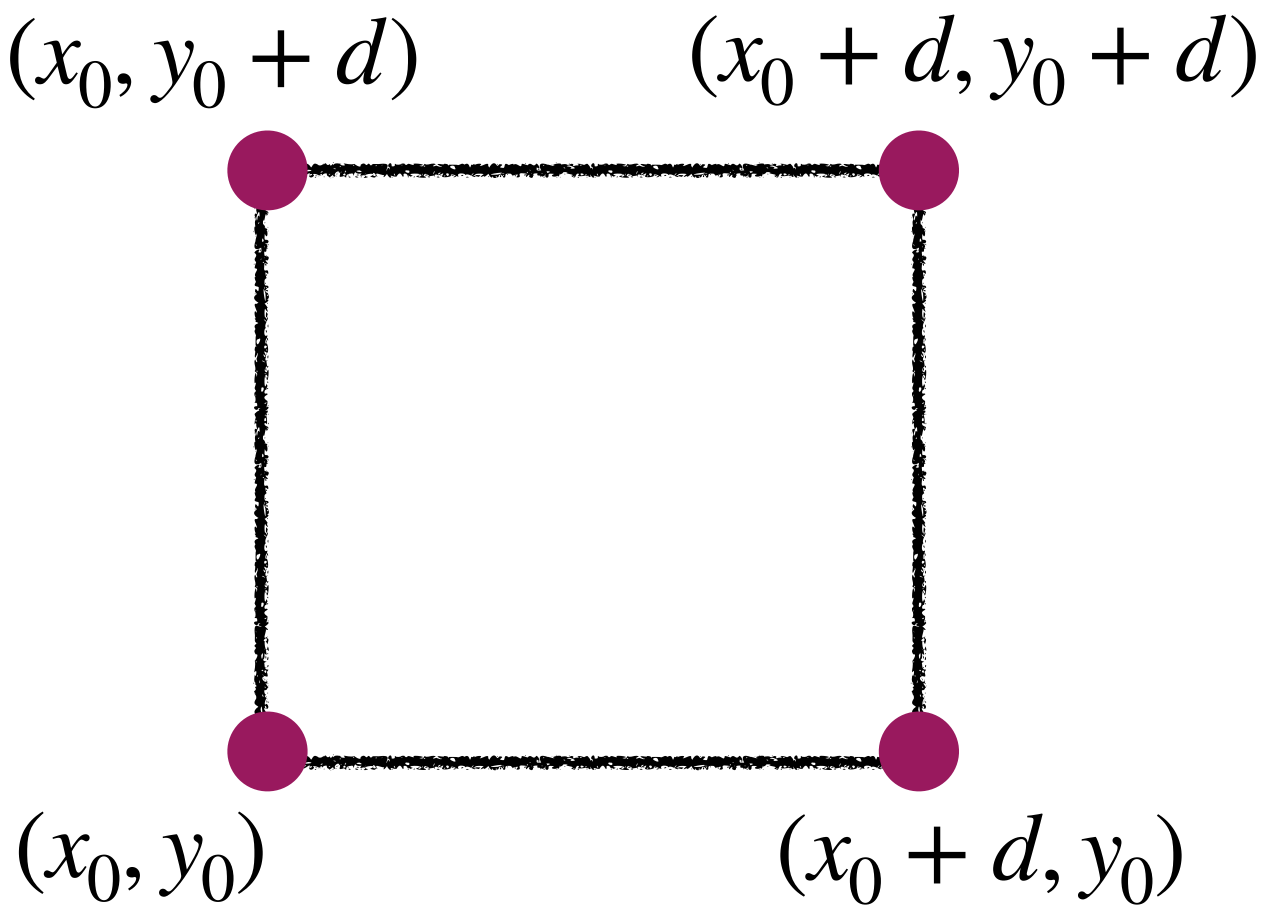

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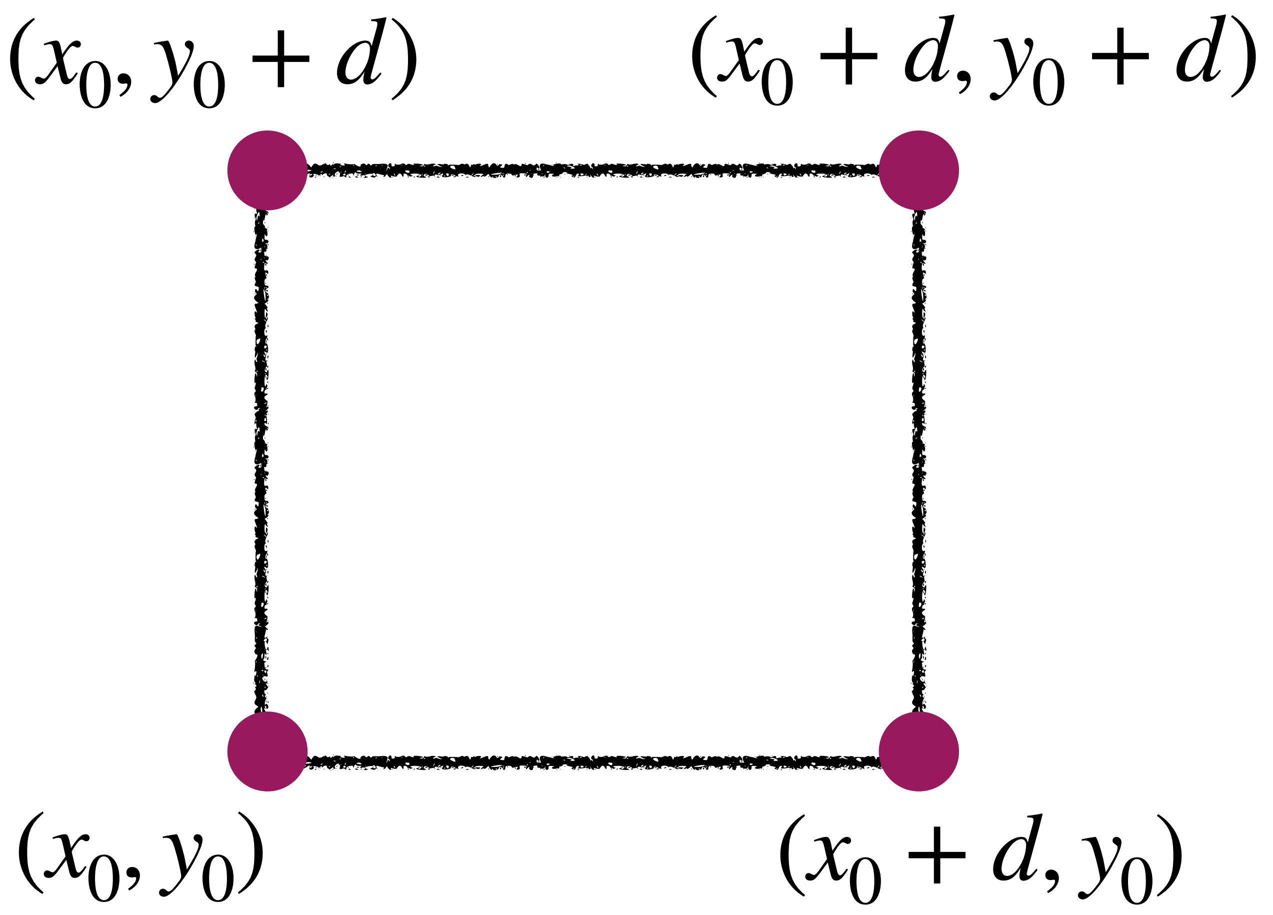

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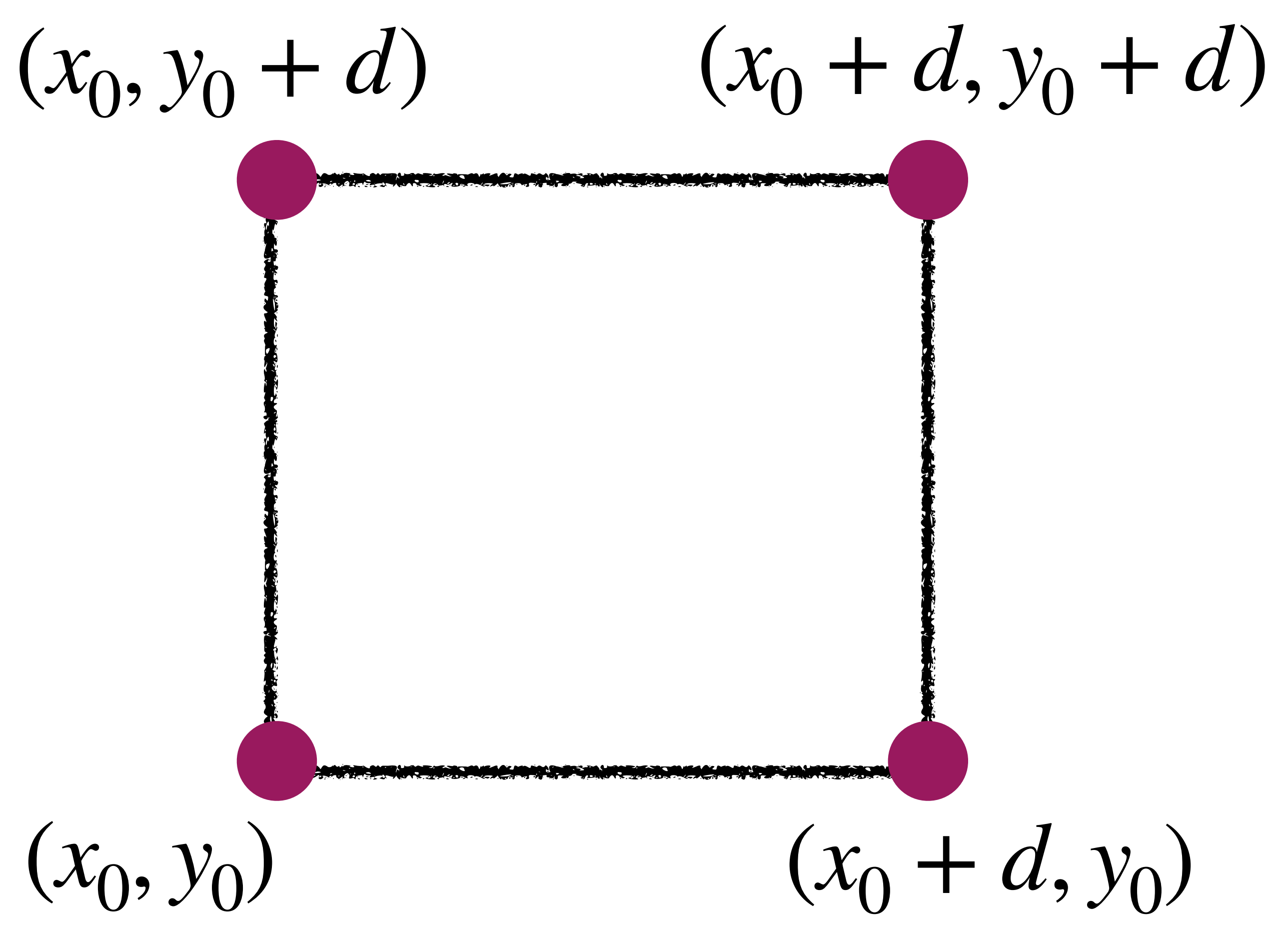

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- Roughly, Any coordinate $i \in [n]$ (with $d_i = 1$) looks like the GHZ support.

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- Uses ideas from 3-AP free sets [KM23], and graph-counting [FHHK24].

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Lemma 1: Above is true if all of E, F, G are algebraically spread.

- Uses ideas from 3-AP free sets [KM23], and graph-counting [FHHK24].

Lemma 2: We can write $E \times F \times G \approx_Q \bigsqcup_i E_i \times F_i \times G_i$ with each E_i, F_i, G_i being algebraically spread.

- Generalizes uniformization strategy in corner-free sets [JLLOS25].

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- Powerful generalization of Fourier uniformity ($r = 1$).

Our Results and Open Problems

Theorem [LLM26]: Let $\mathscr{G}$ be a GHZ-supported game with val$(\mathscr{G}) < 1$. Then,

$$\text{val}(\mathscr{G}^{\otimes n}) \leq \exp(-n^c).$$

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Future Directions:

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Future Directions:

- Truly exponential bounds?
- First result holds if distribution is not uniform, but second does not.
- Other applications of techniques: additive combinatorics? communication complexity?…

Thank You!

Questions?