Faux-determinism

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Overview

- Define faux deterministic algorithms in decision tree model which
- roughly characterizes “computationally indistinguishable” from deterministic

Introduction Some preliminary definitions

Motivation Search problems and why randomness could be different for them

Definition Defining faux deterministic model

Construction Building the construction for FIND1 problem

Introduction

Some preliminary definitions

Decision tree model

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Goal: compute $f : \{0,1\}^* \to \{0,1,\perp\}$

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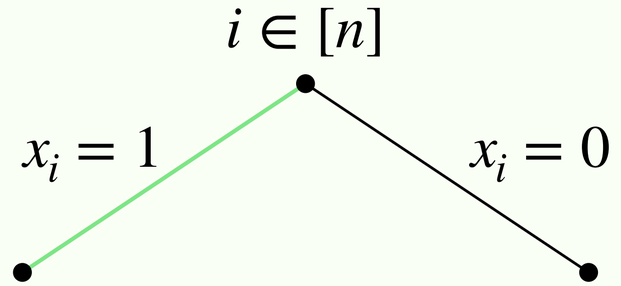

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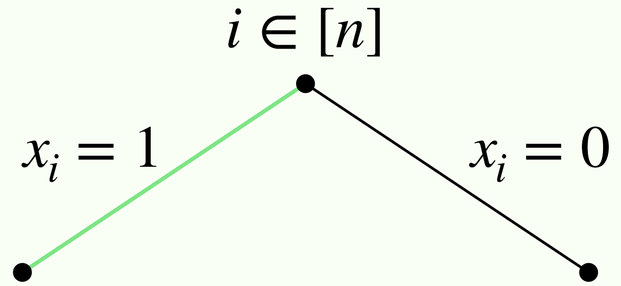

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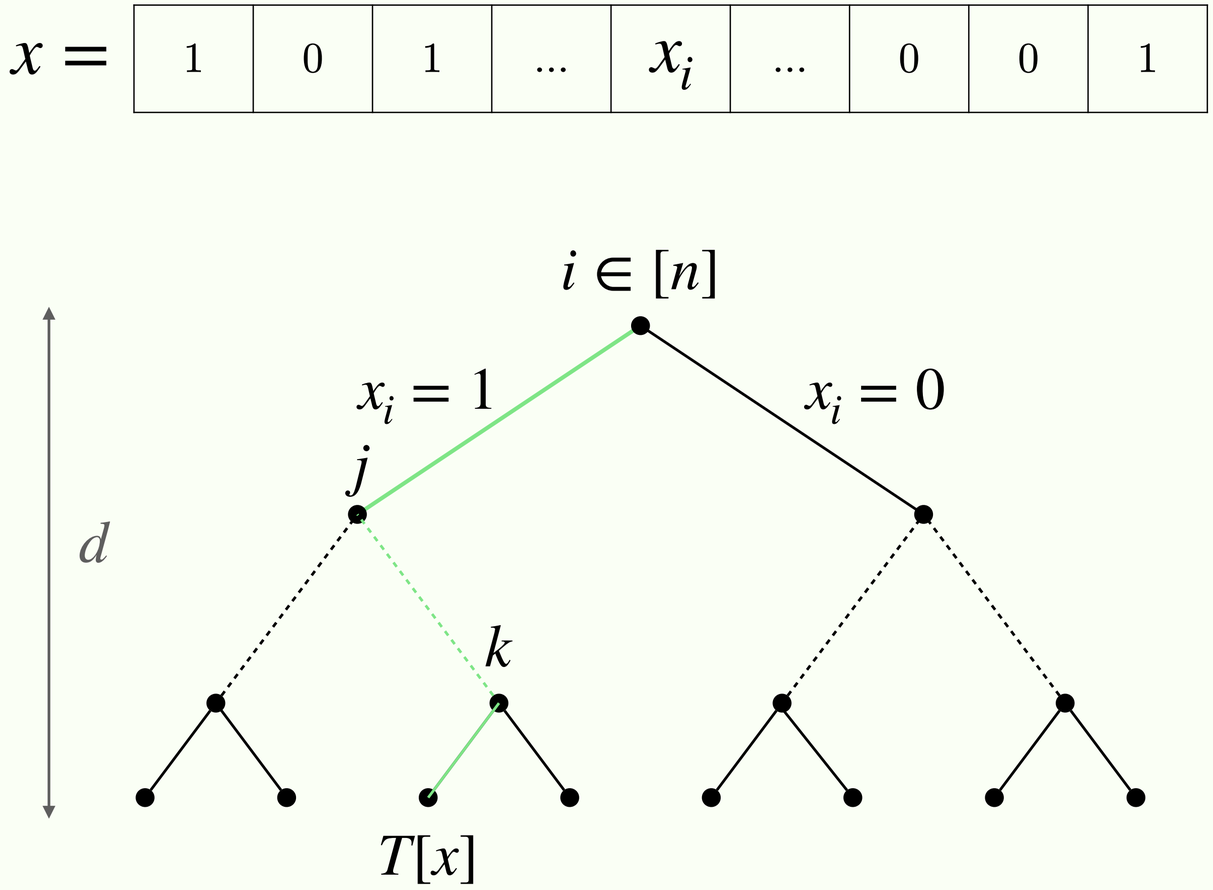
- output $T[x] = \text{f(x)}$



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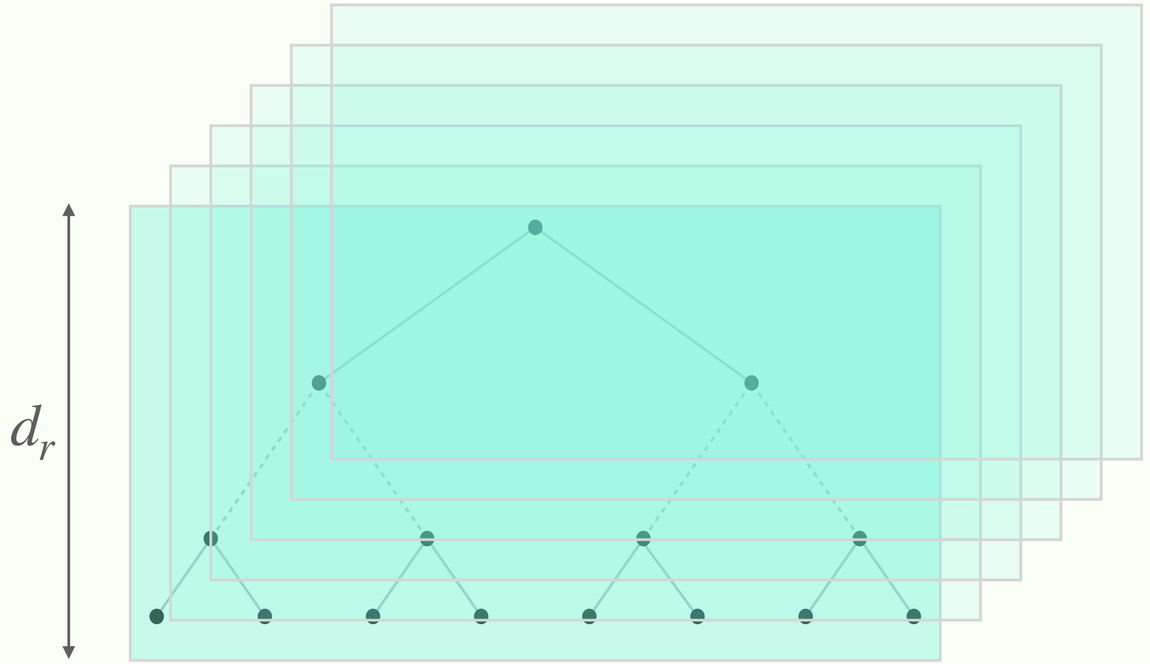

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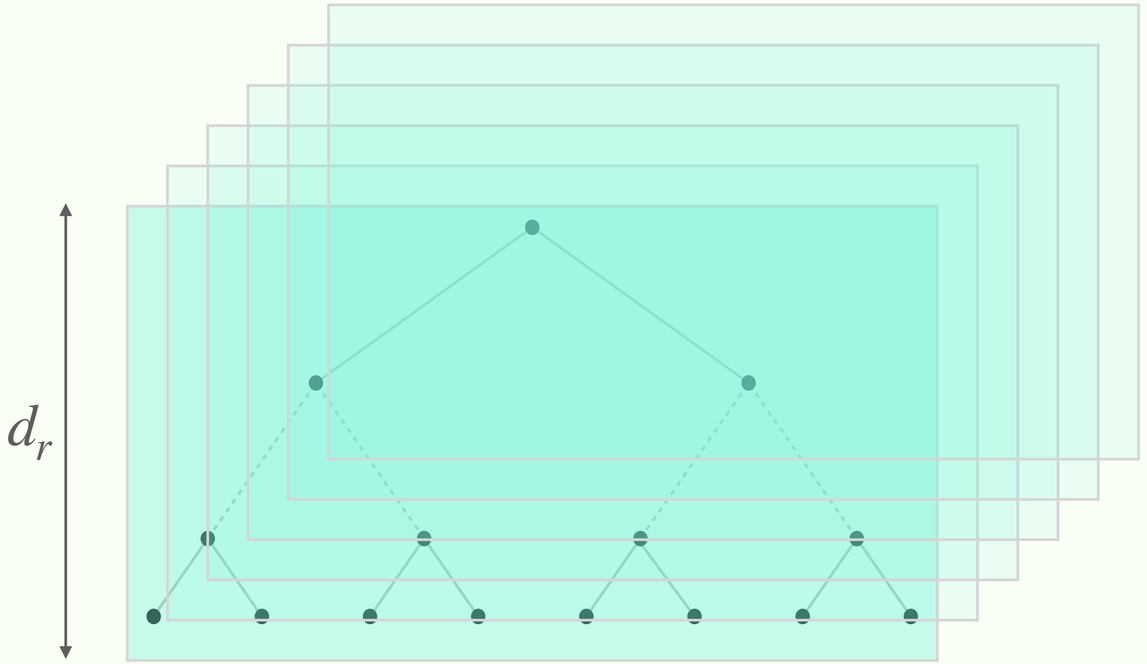

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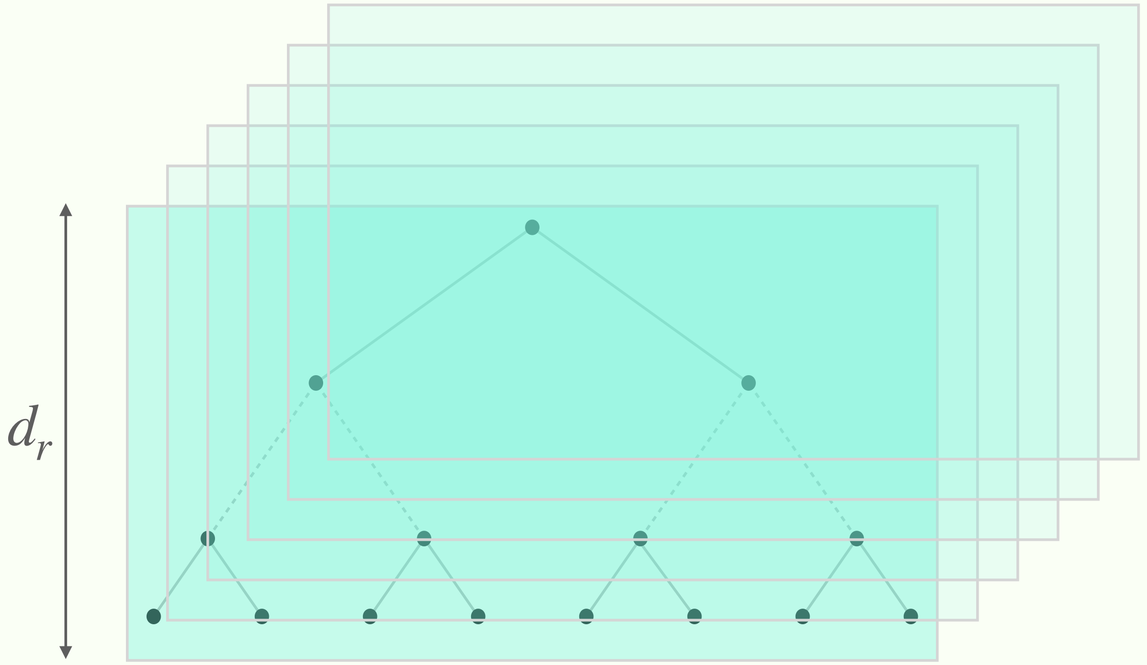

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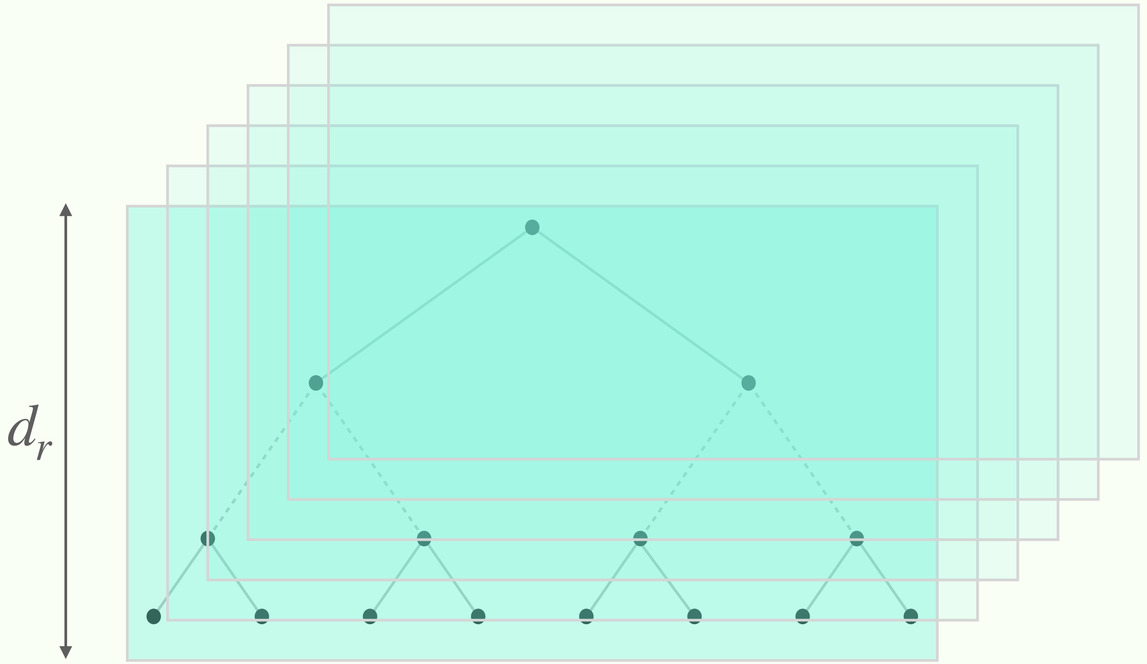

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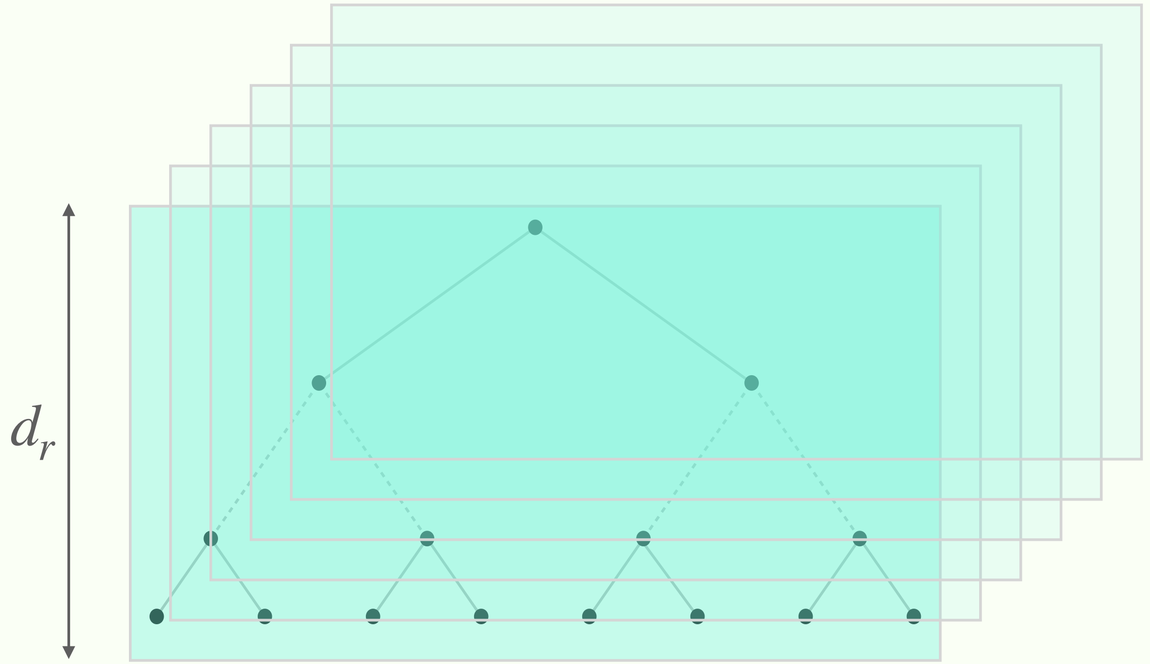

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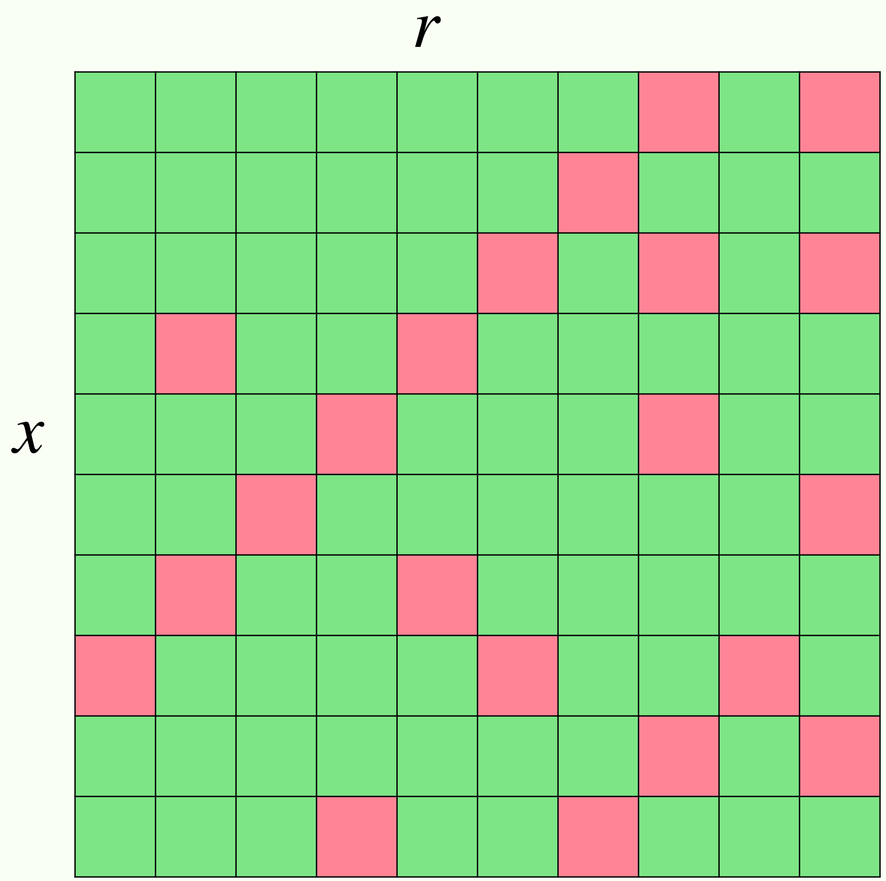

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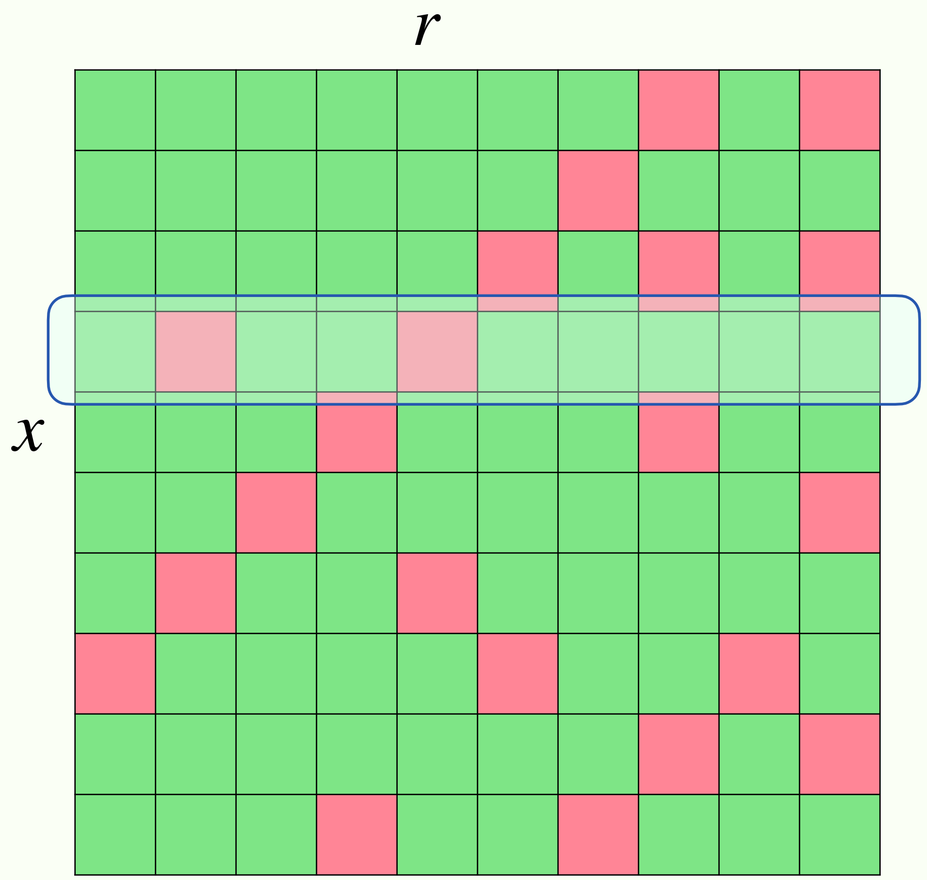

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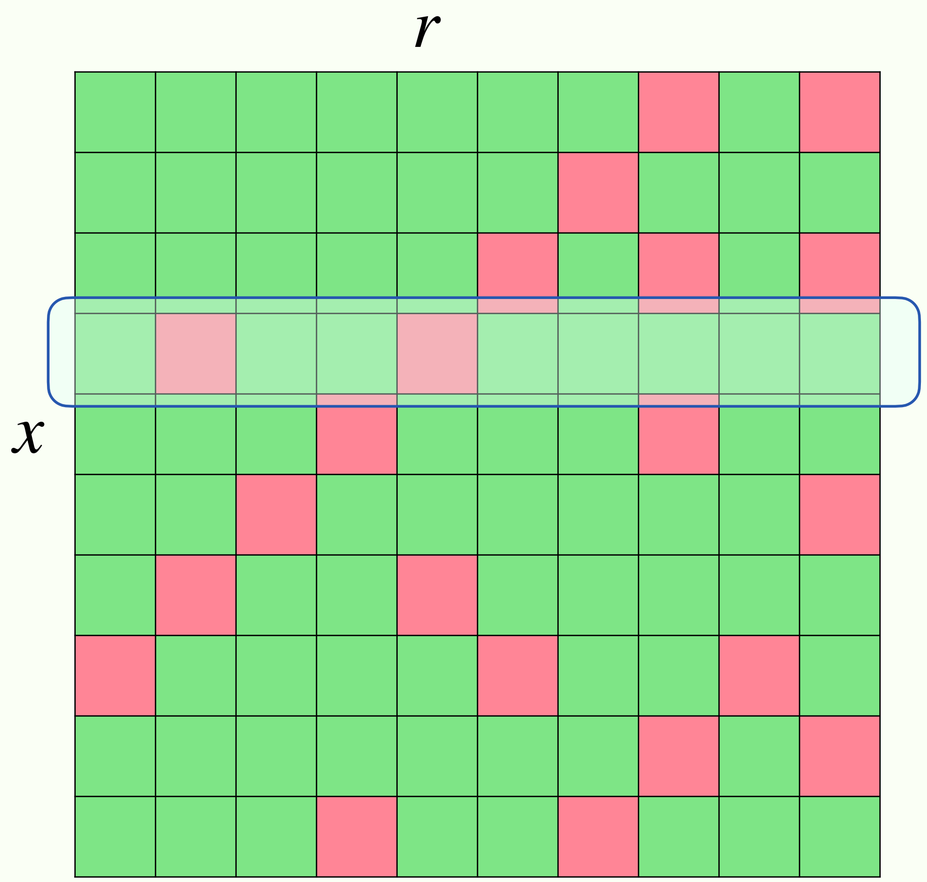

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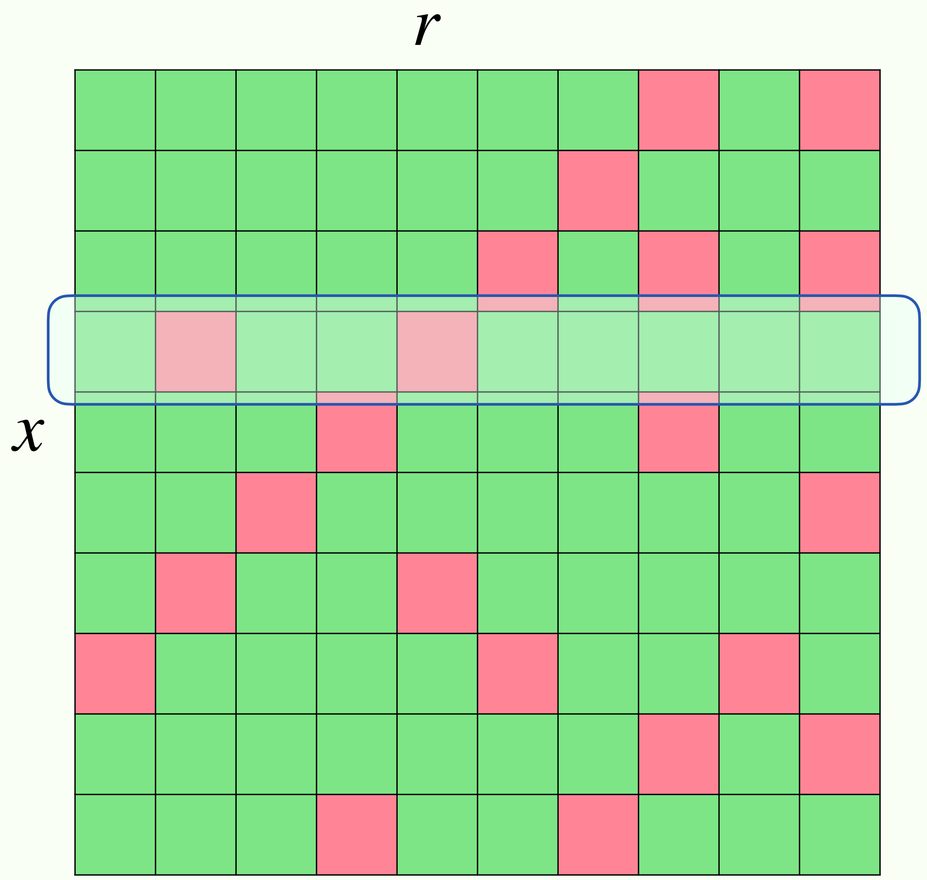

- Error amplification to $\epsilon = 1/\text{quasipoly}(n)$
- indistinguishable from deterministic algorithm

Computational distinguishability from deterministic algorithms

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Algorithm
$\mathscr{T} = \{T_r\}$

r

x

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Computational distinguishability from deterministic algorithms

Adversary

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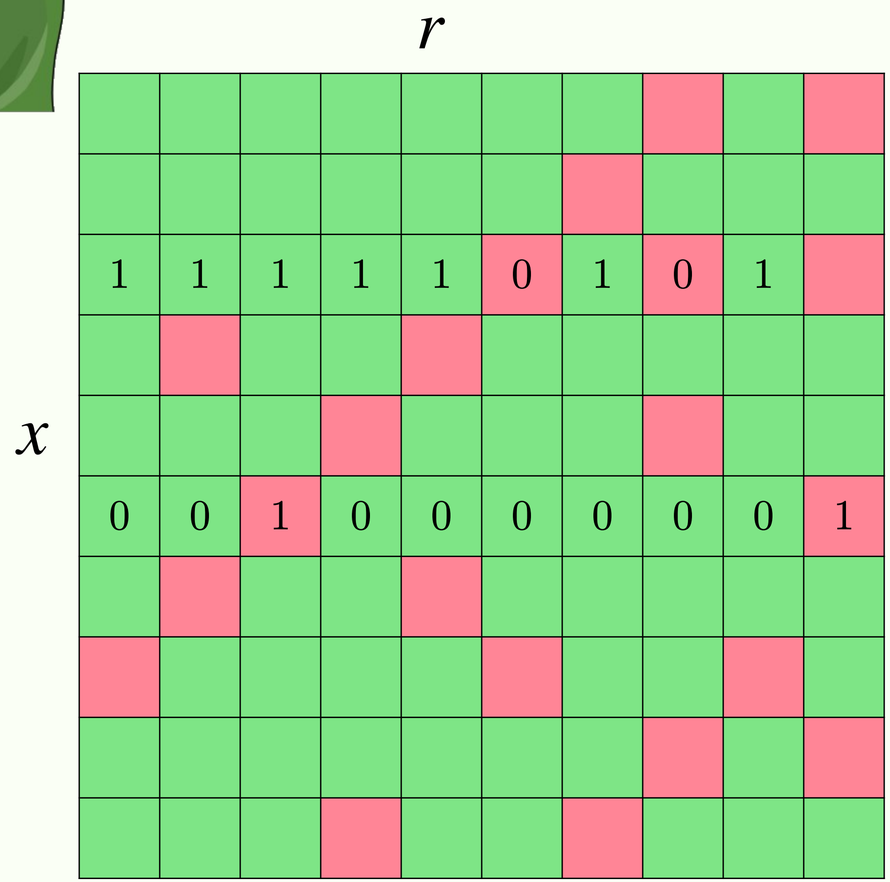

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What is $f(x_1)$?

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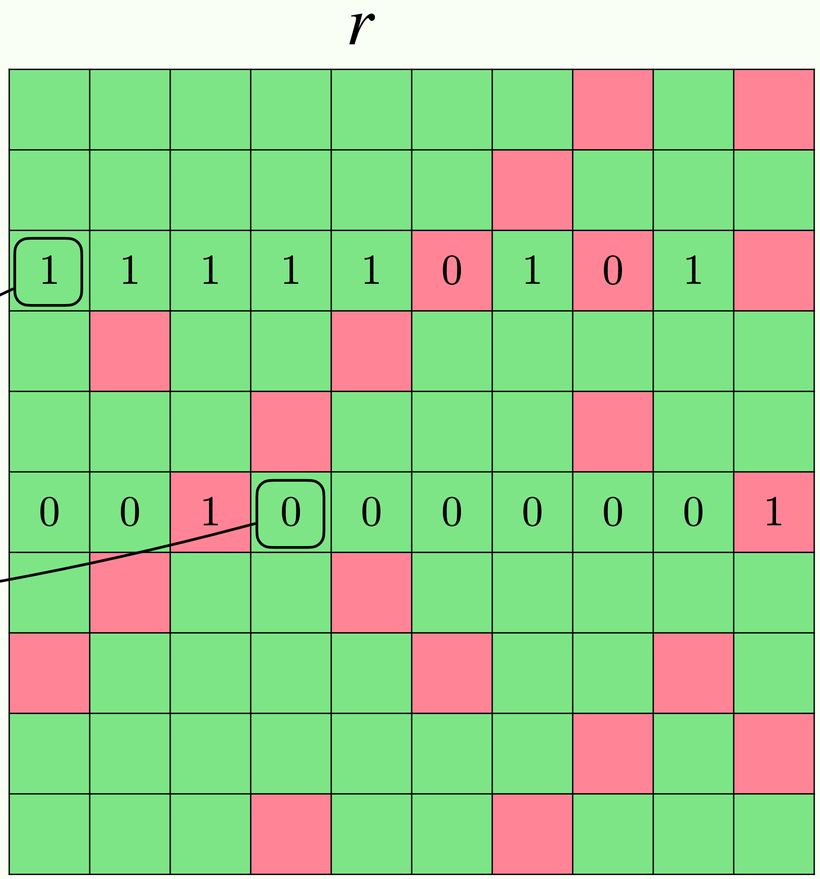

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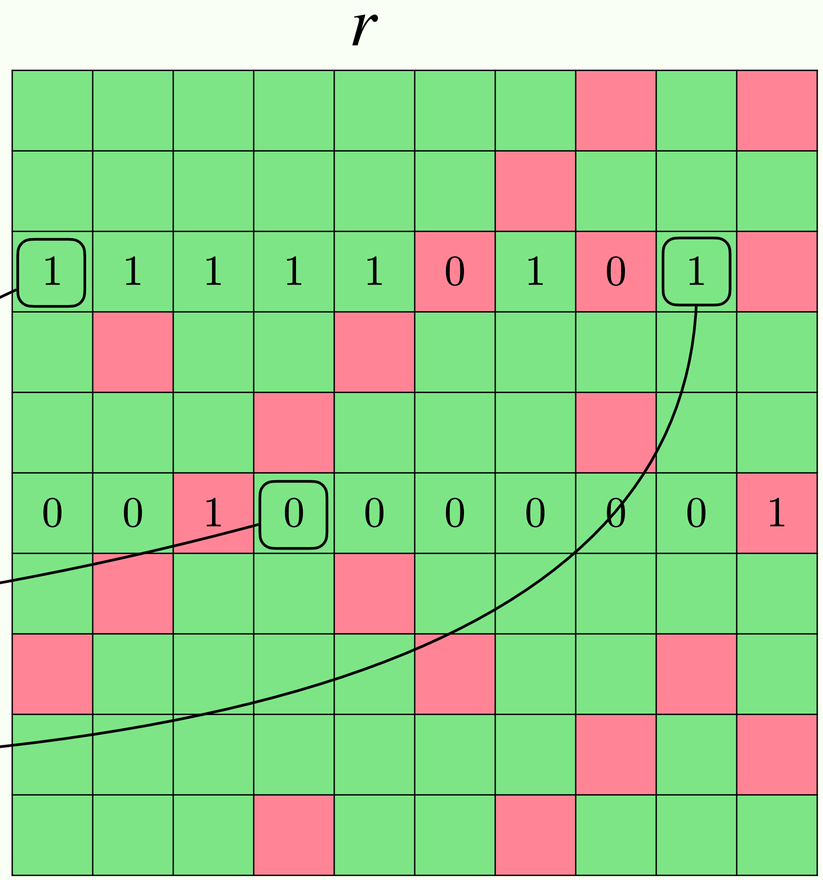

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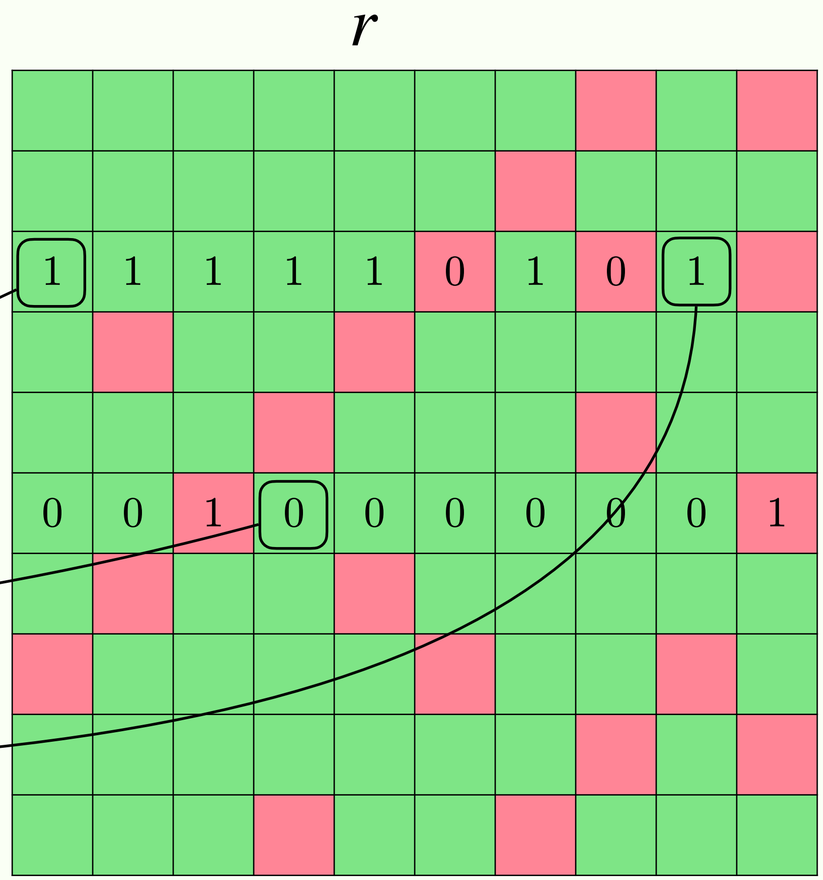

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Success: $\text{poly}(n) \times \dfrac{1}{\text{quasipoly}(n)} \ll 1$

Motivation

Search problems and why randomness could be different for them

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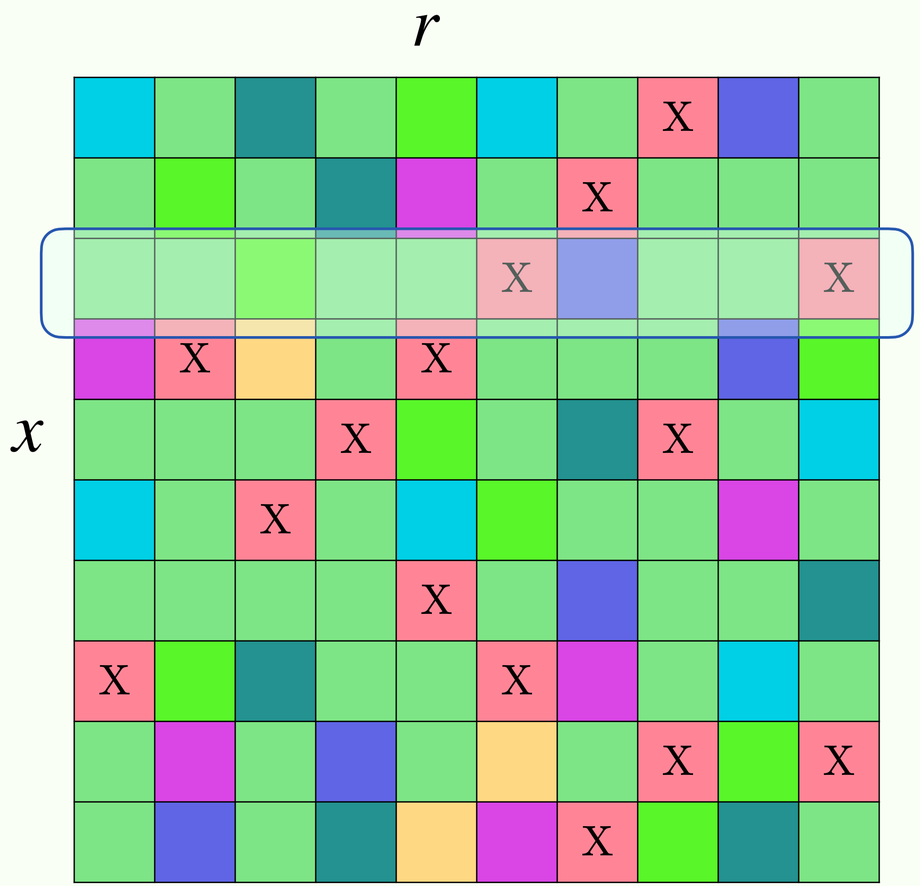

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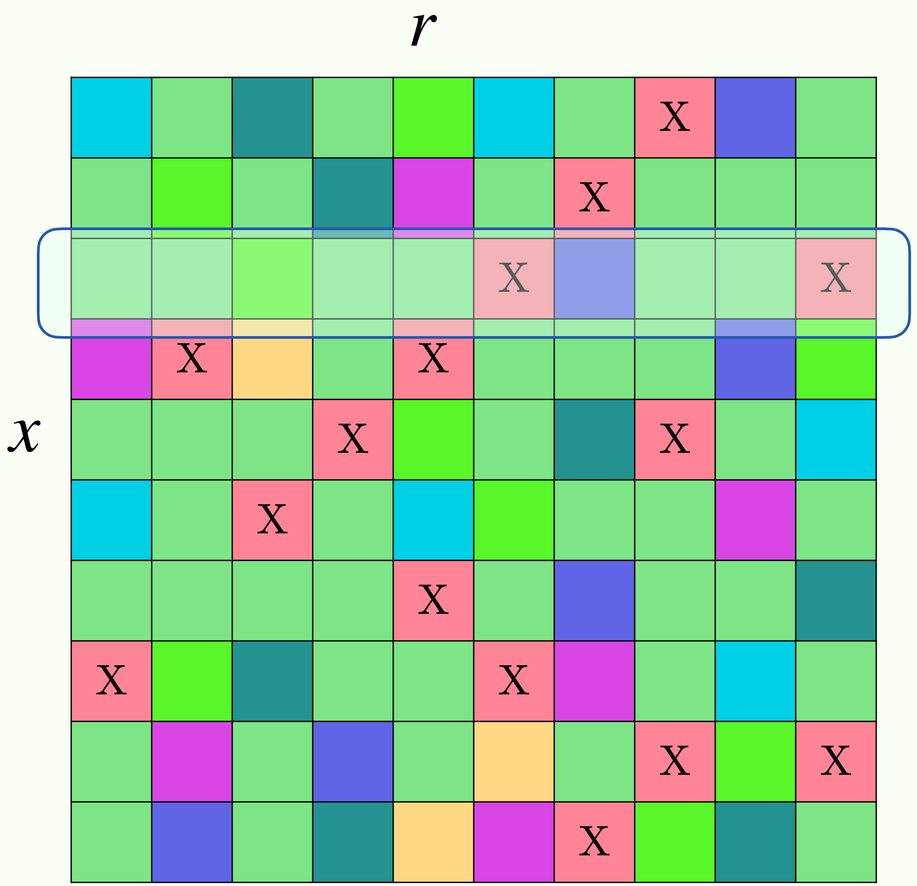

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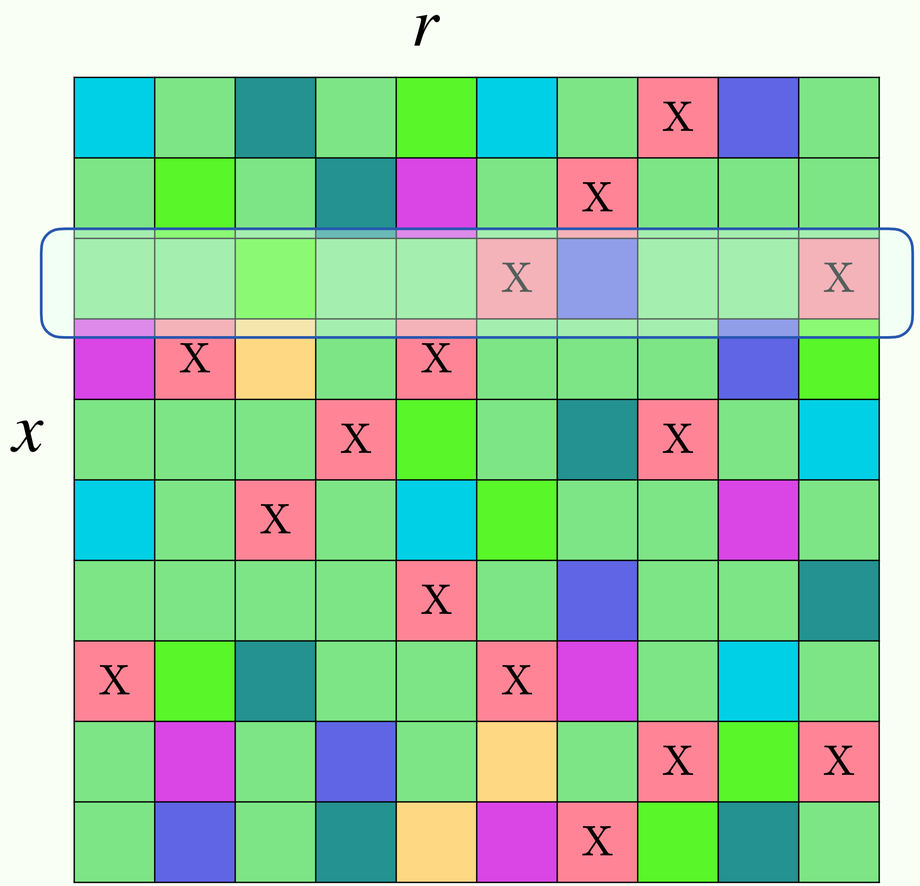

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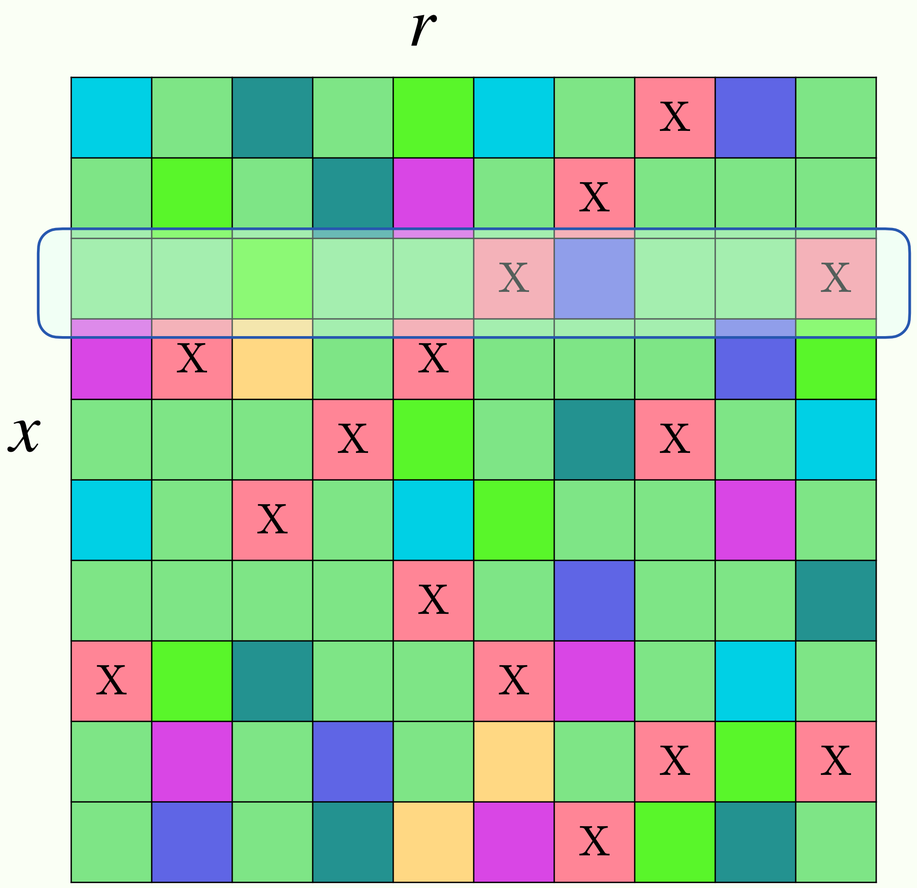

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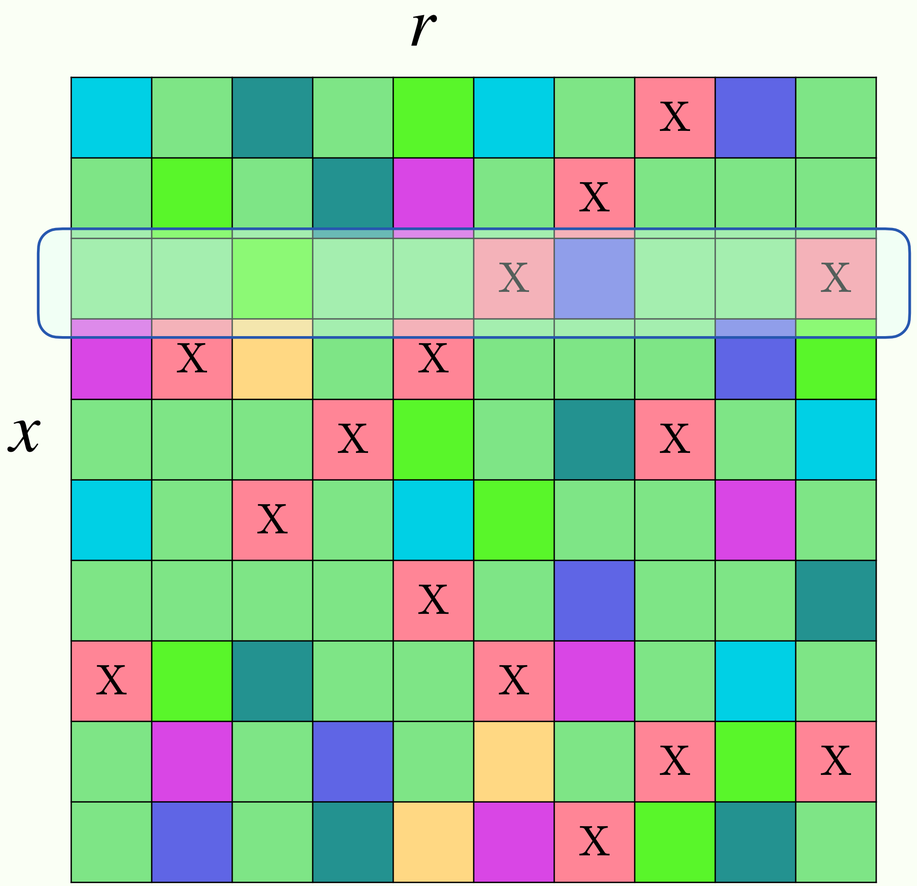

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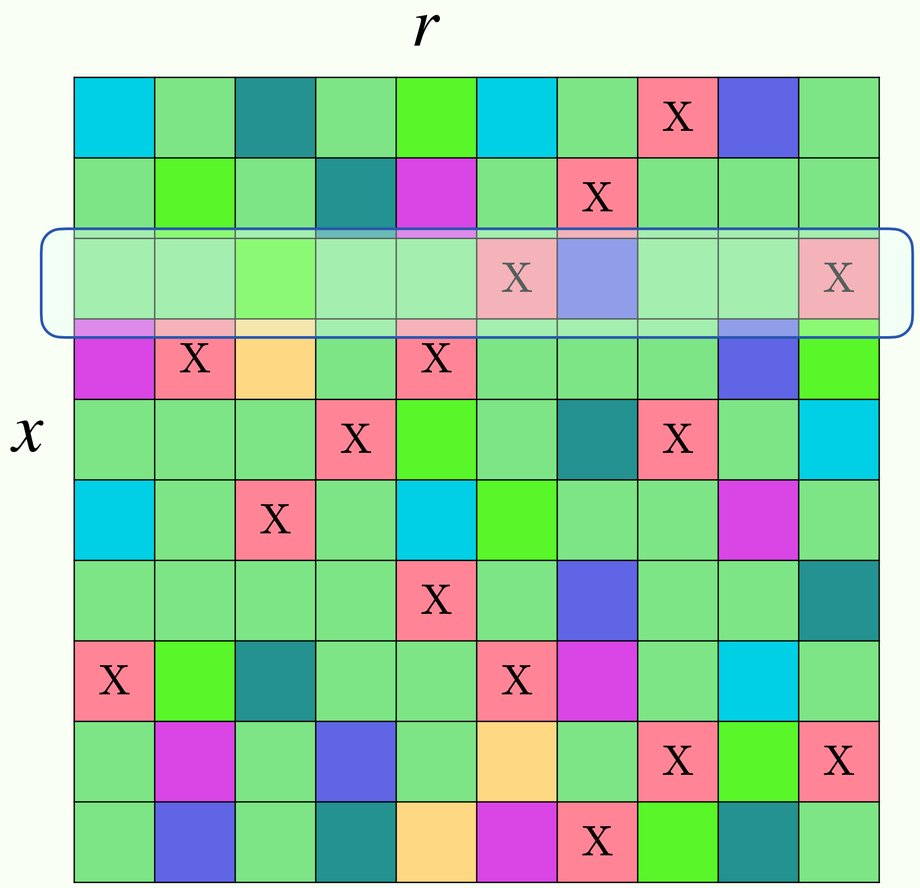

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(different outputs for the same input)

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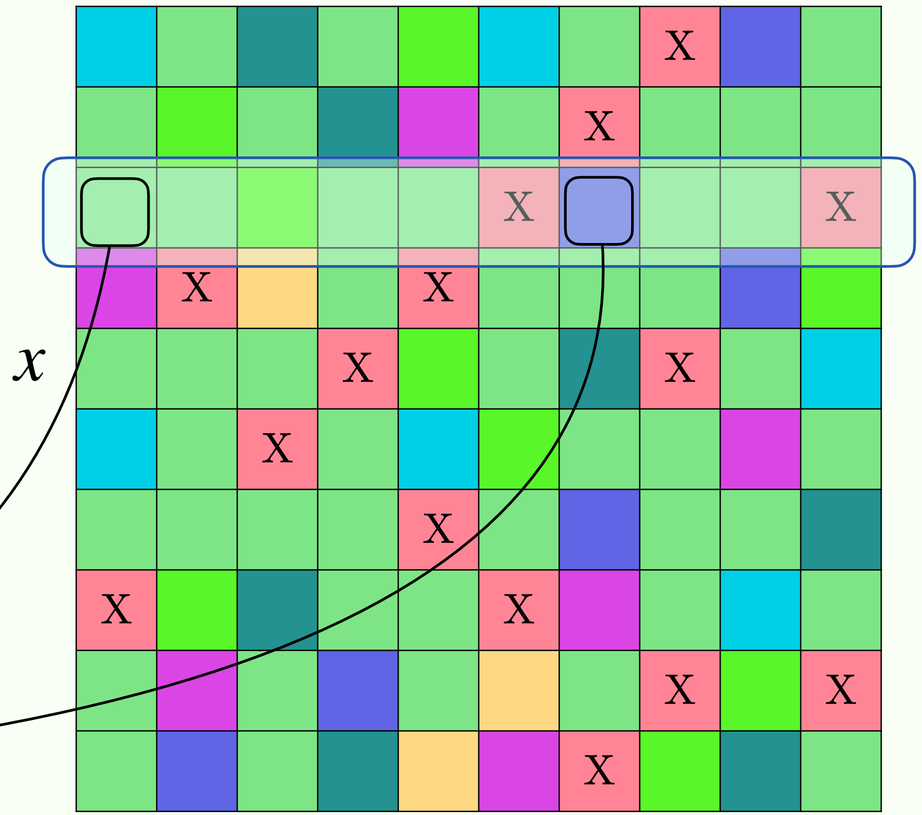

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Randomized tree $\mathcal{T} = \{T_r : r \sim R\}$:

- input $x \in \{0,1\}^n$ and seed r
- Correctness: $\Pr_r[T_r[x] \in S(x)] \geq 2/3$
- $\mathsf{FBPP}^{\mathsf{dt}} = \{S : \exists$ efficient $\mathcal{T}$ computing $S\}$

- ~~Error amplification~~ ←fixed by verifiability
- $S \in \mathsf{FBPP}^{\mathsf{dt}} \cap \mathsf{FNP}^{\mathsf{dt}} \Rightarrow \epsilon = 1/\text{quasipoly}(n)$
- ~~indistinguishable from deterministic algorithm?~~

What is $f(x_1)$?

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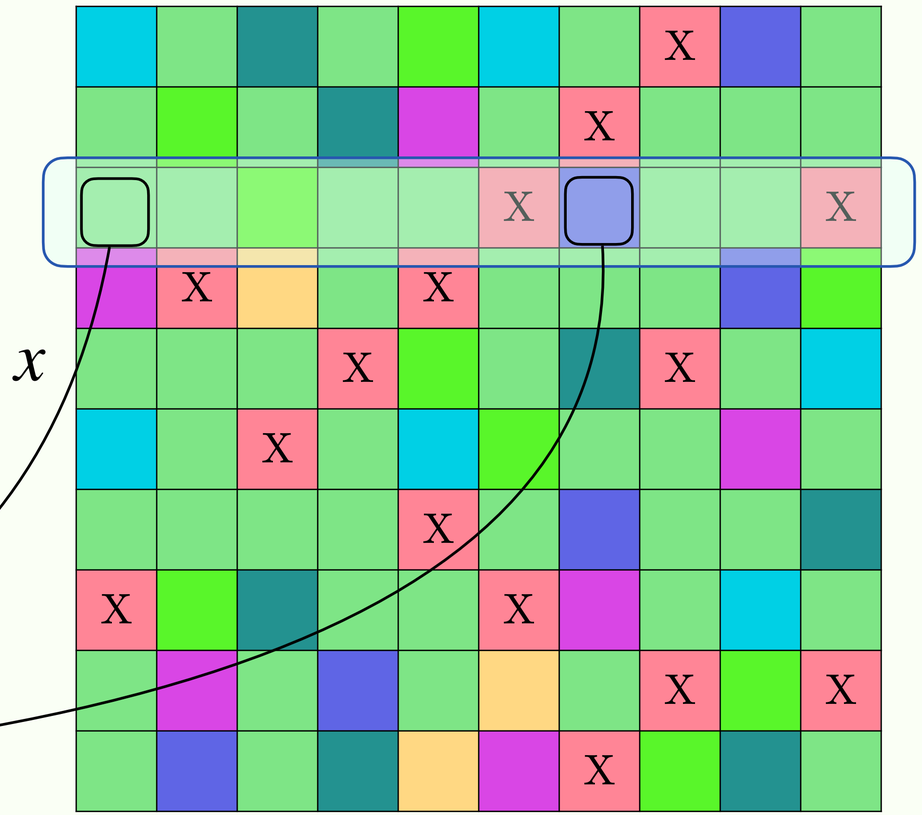
Problem: not necessarily consistent answers

(different outputs for the same input)



Pseudo-deterministic algorithm?

Pseudo-deterministic algorithms for search problems [GG11]

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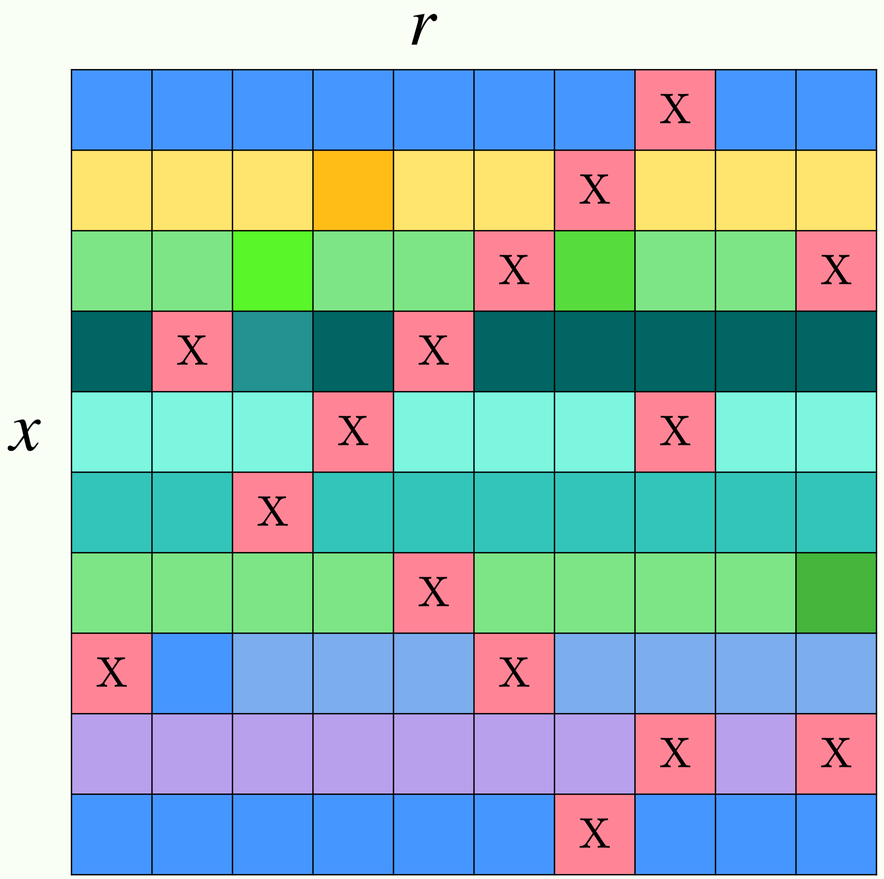

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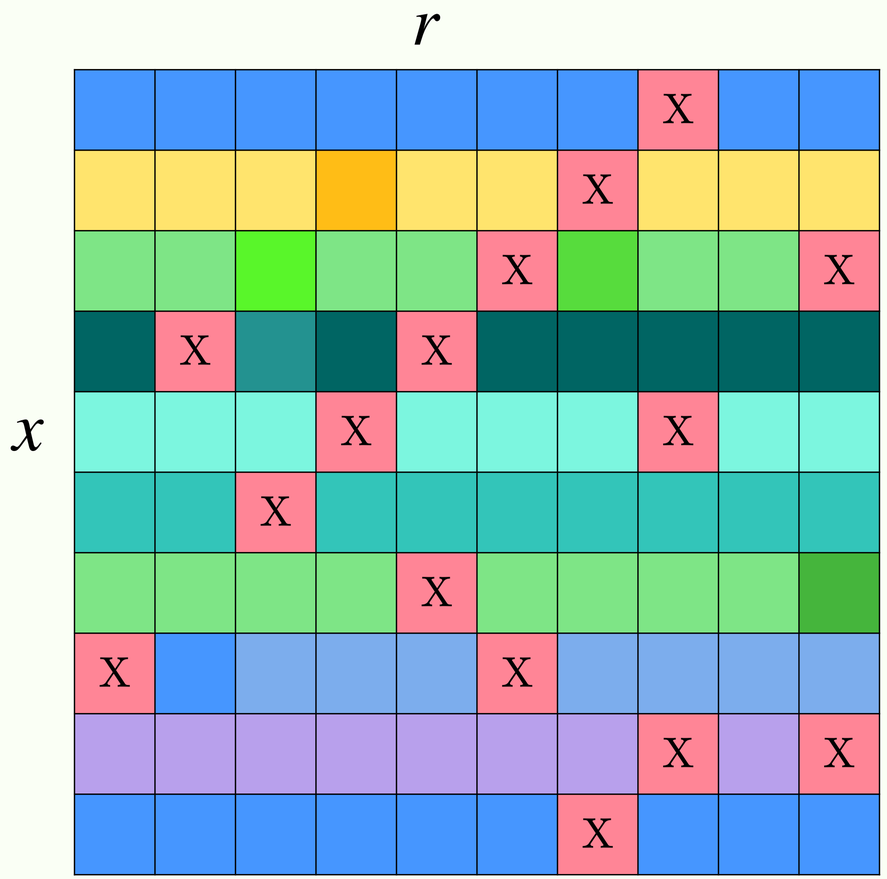

- indistinguishable from deterministic algorithm

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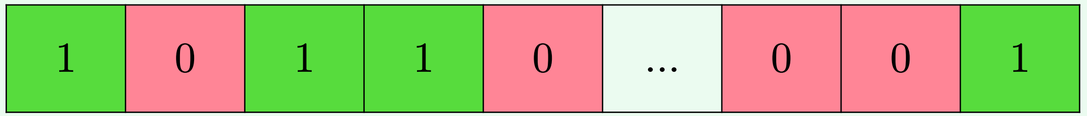

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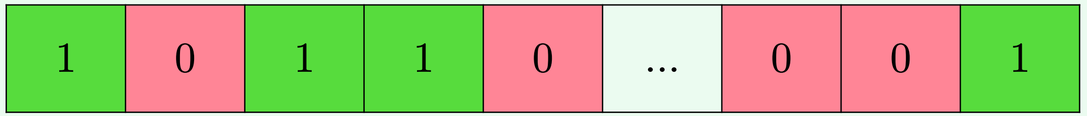

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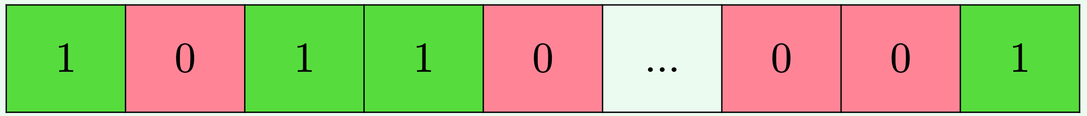

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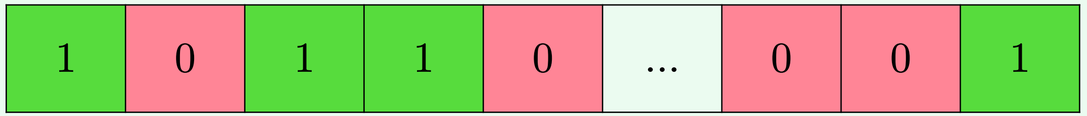

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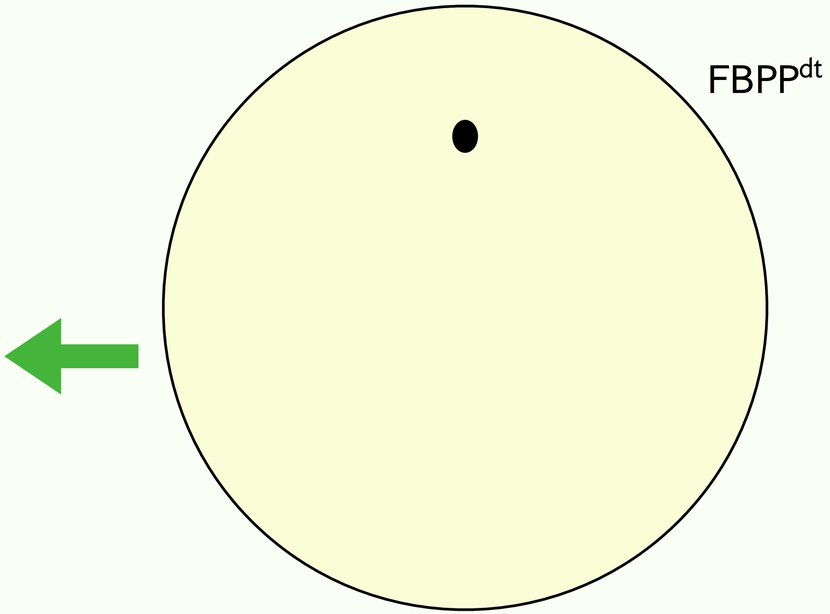

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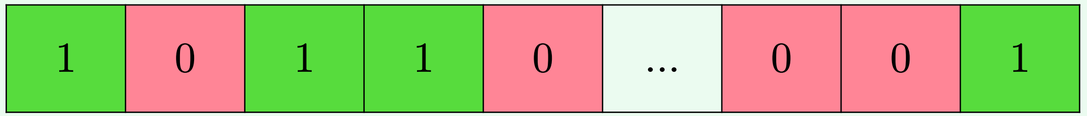

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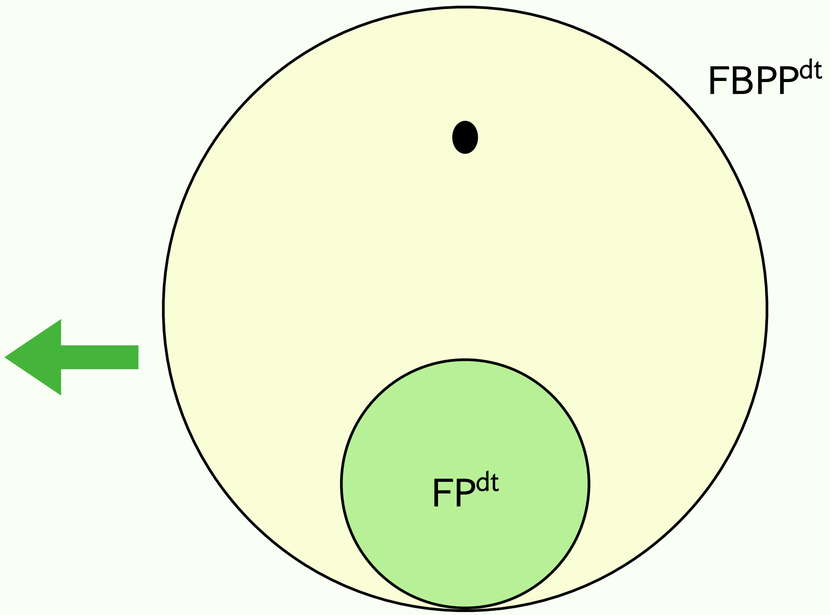

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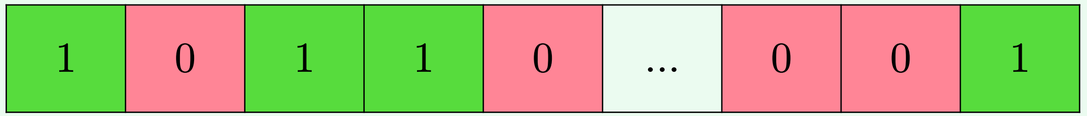

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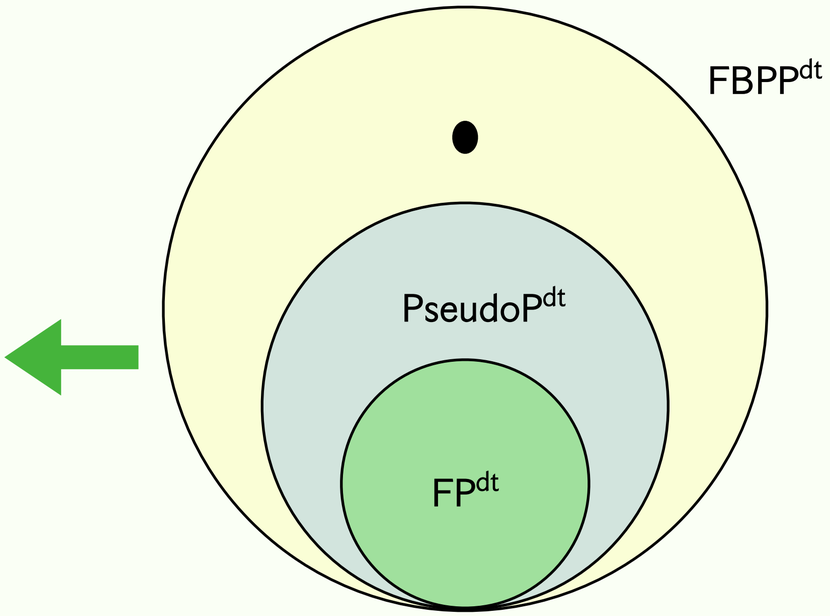

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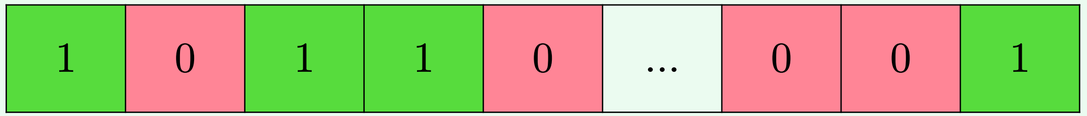

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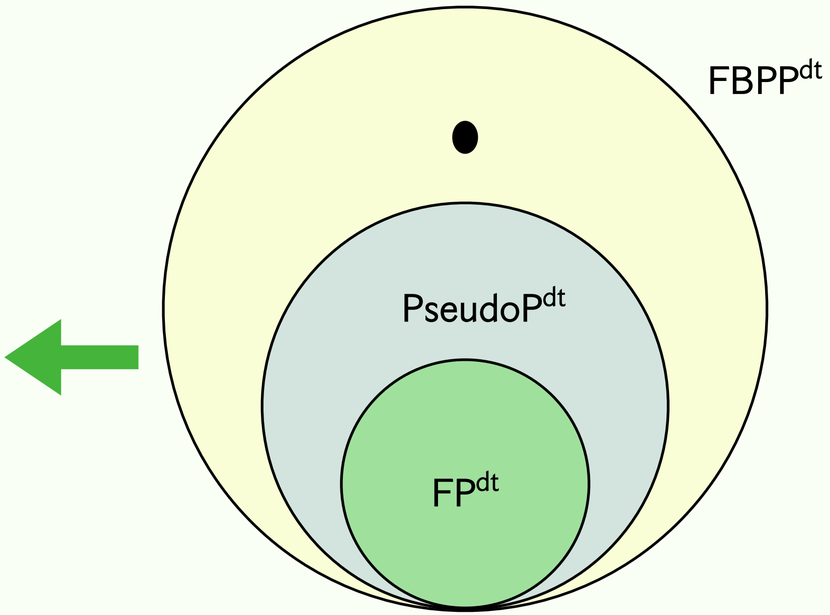
Can we get a cheaper "pseudo"-deterministic algorithm?



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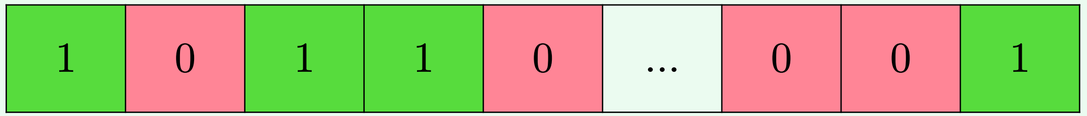

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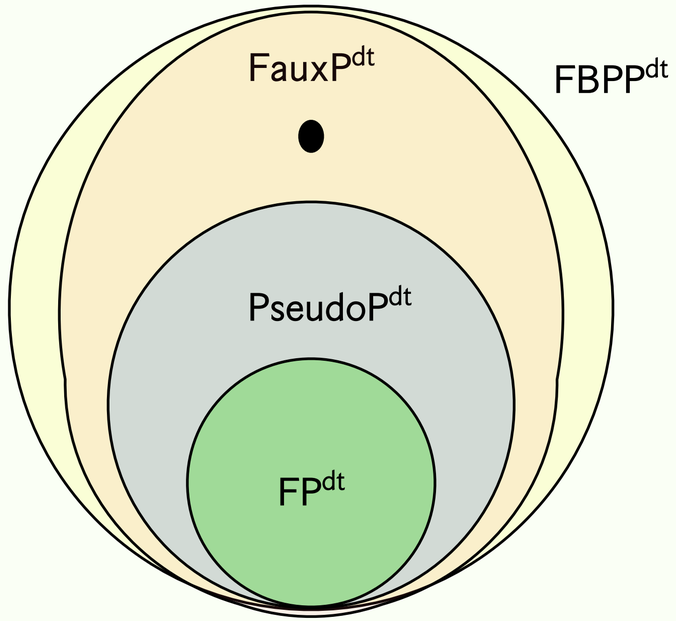

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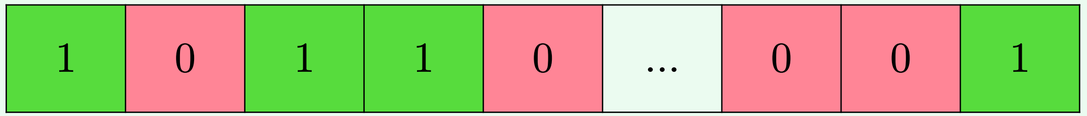

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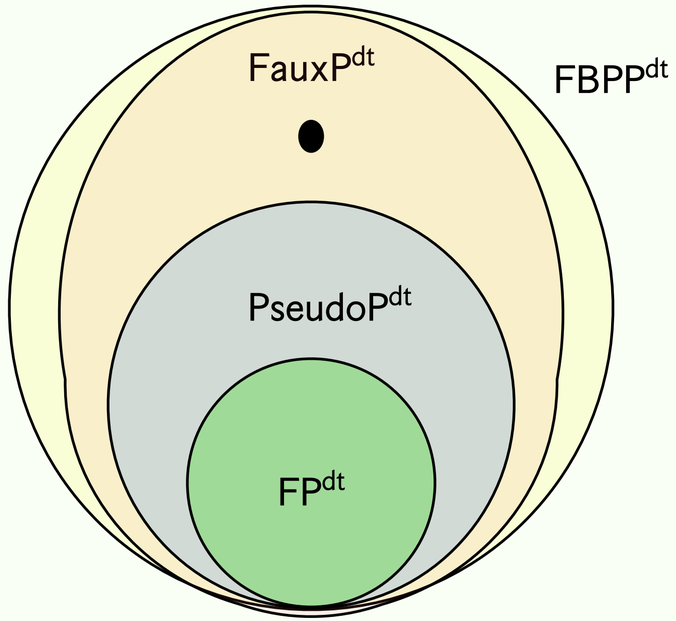

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1	0	1	1	0	...	0	0	1

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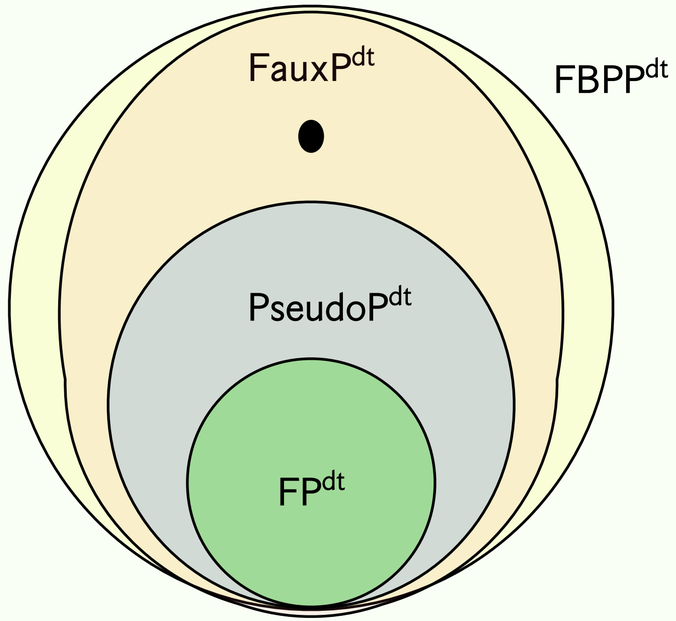
Requires a non-constructive proof?



Faux-determinism

Defining faux deterministic algorithms

Faux-deterministic algorithms for search problems

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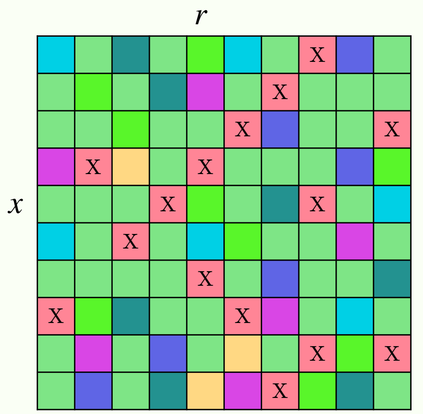
Faux-deterministic tree $\mathscr{T} = \{T_r : r \sim R\}$:



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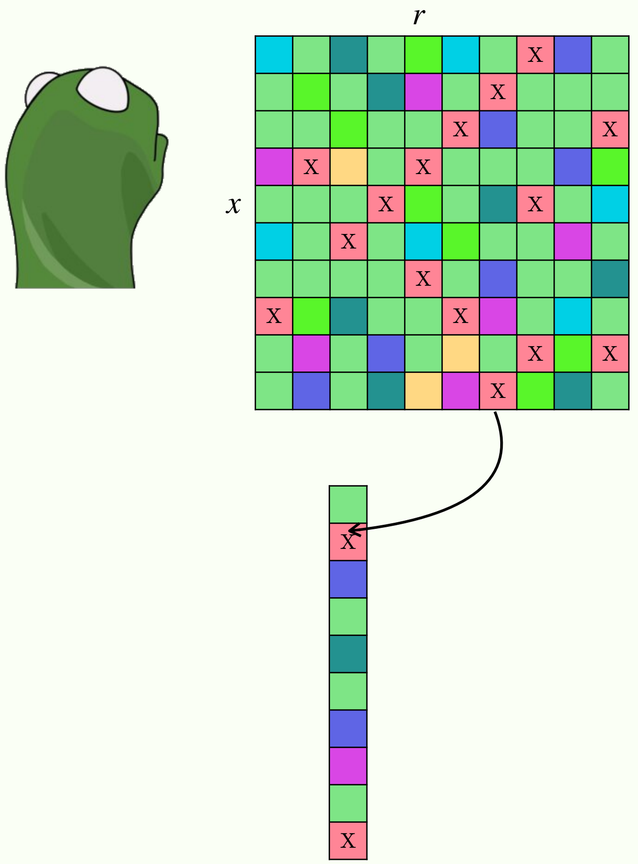

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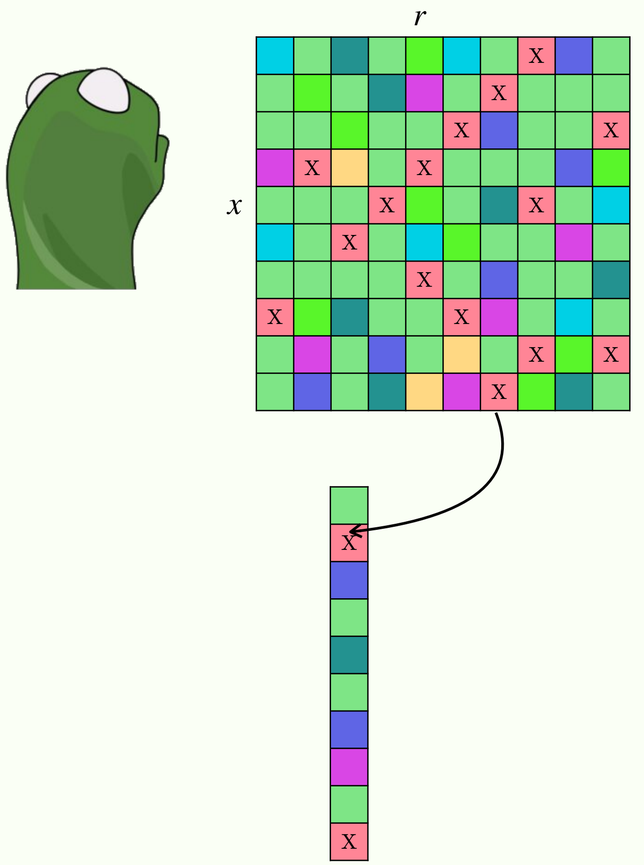

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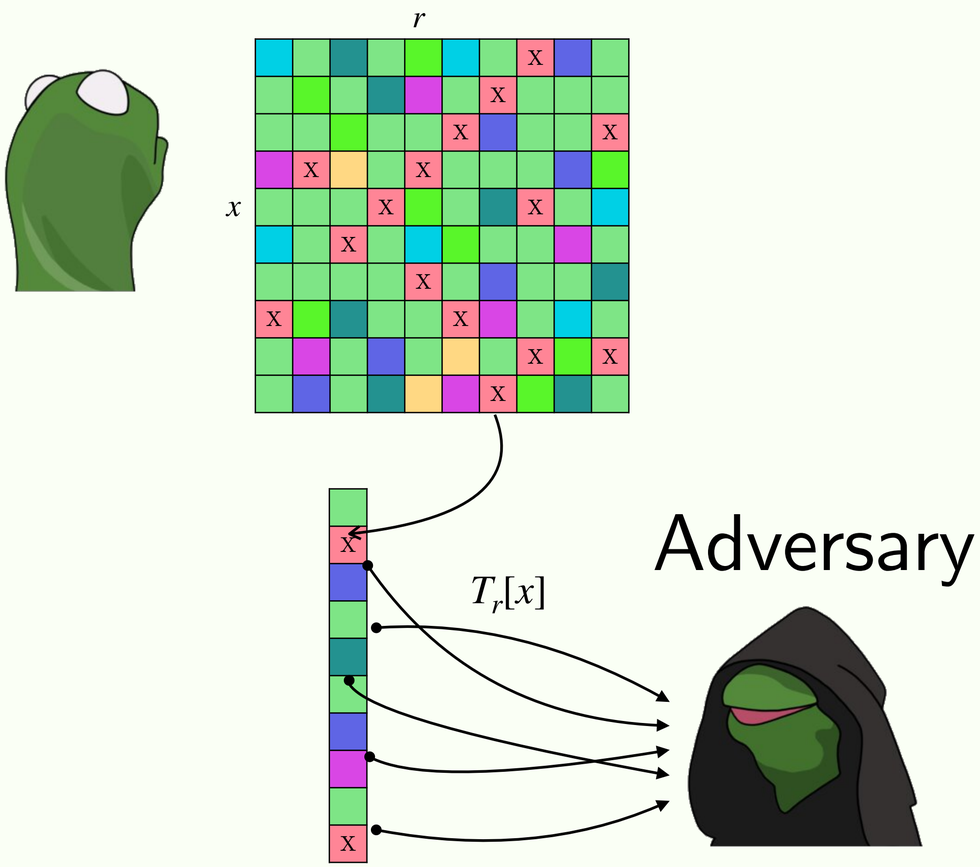

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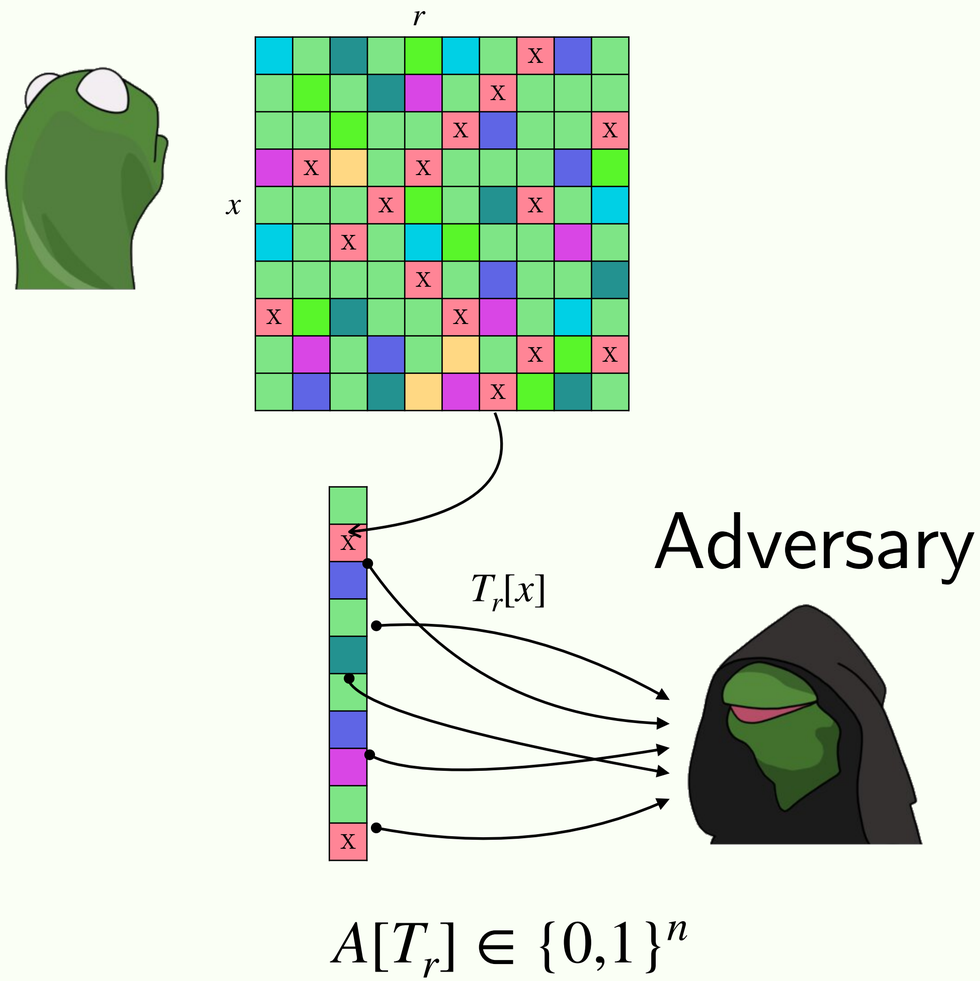

$A[T_r] \in \{0,1\}^n$

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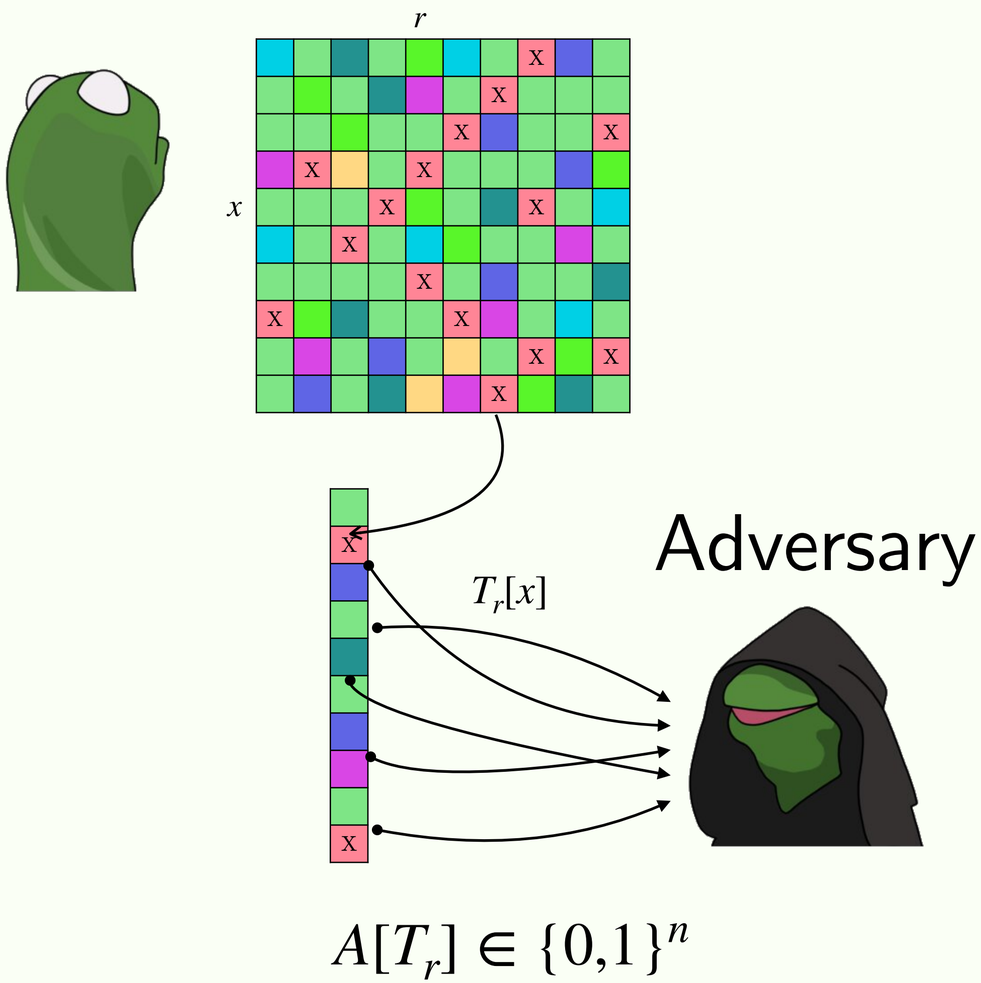

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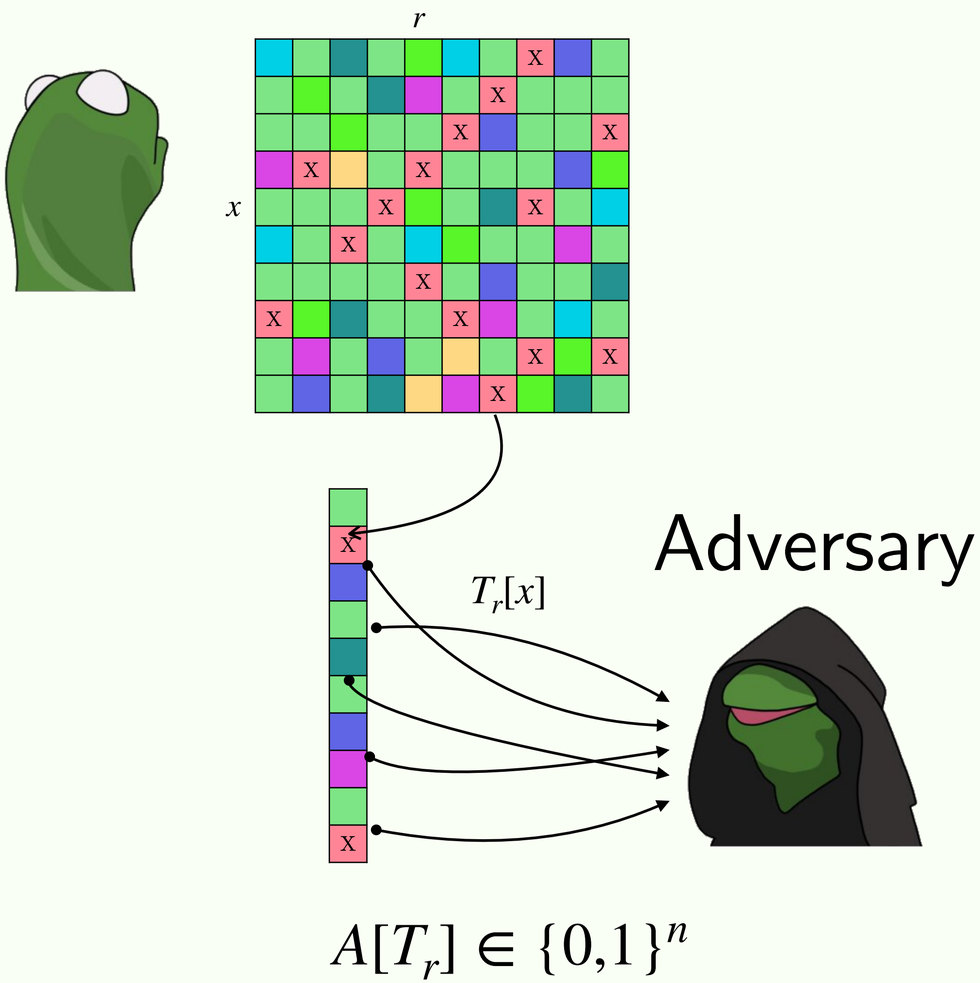

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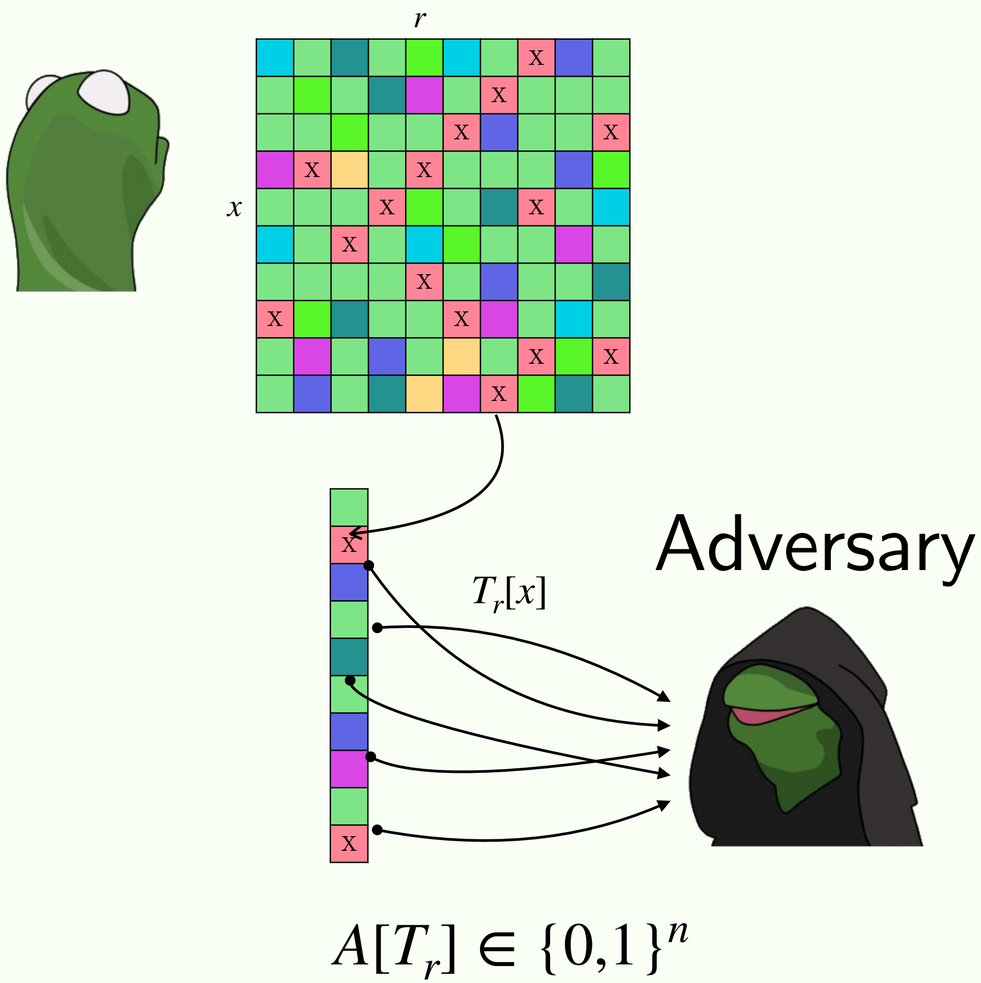

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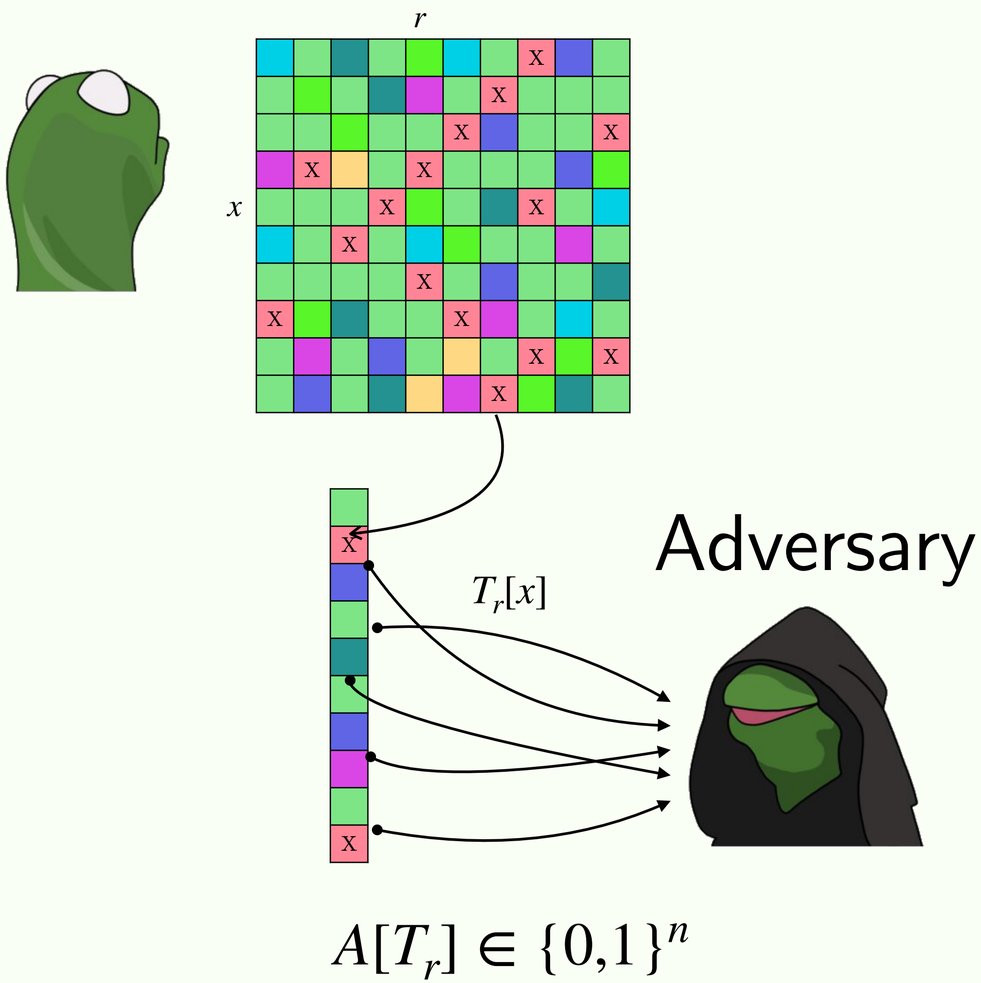
Remark: "set it and forget it" paradigm: the given tree is a fixed deterministic tree



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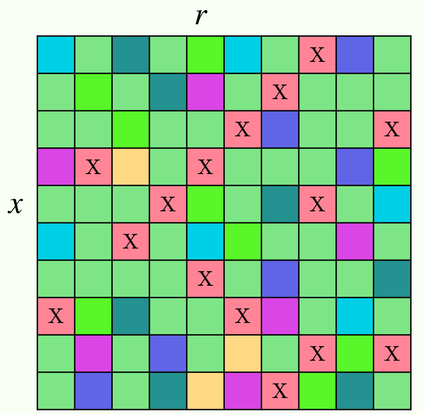

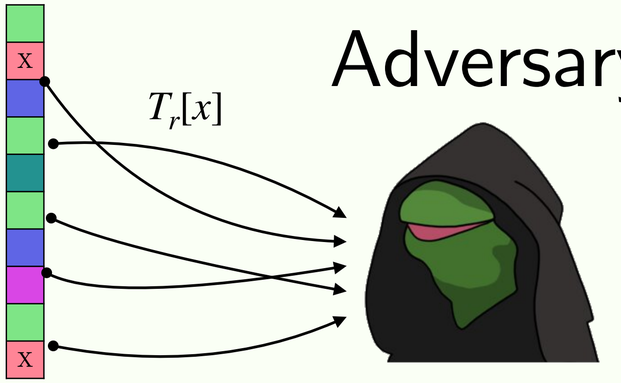

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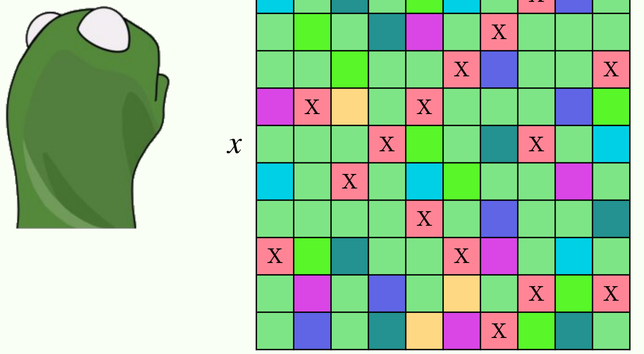

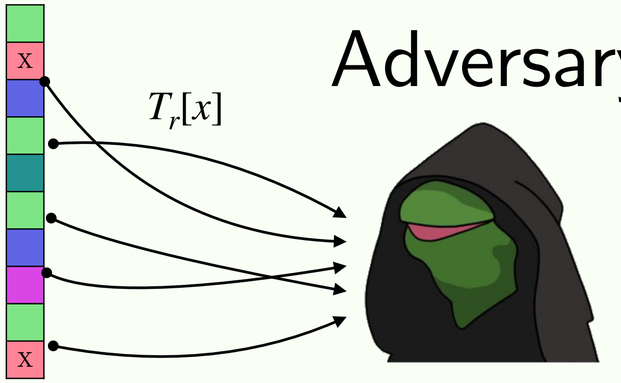

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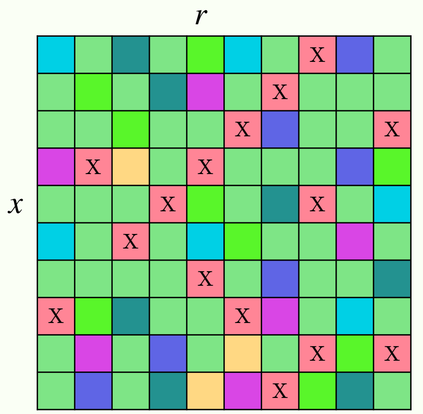

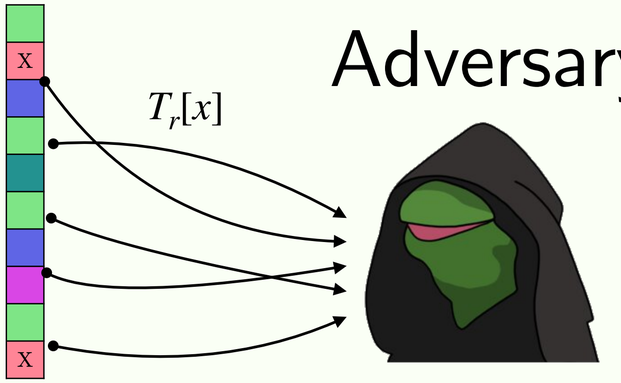

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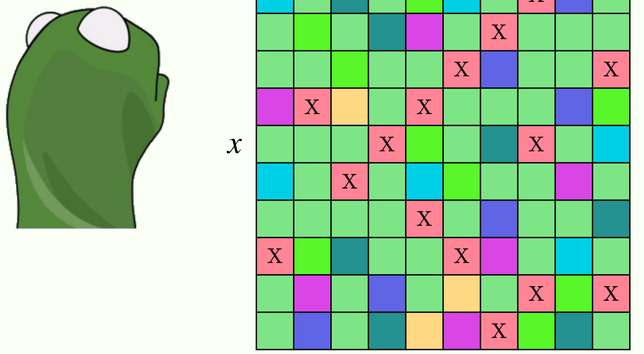

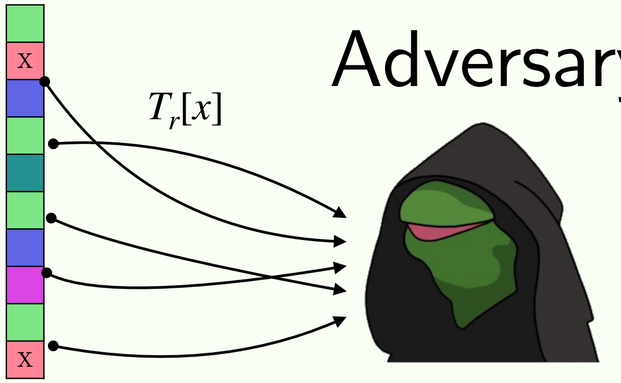

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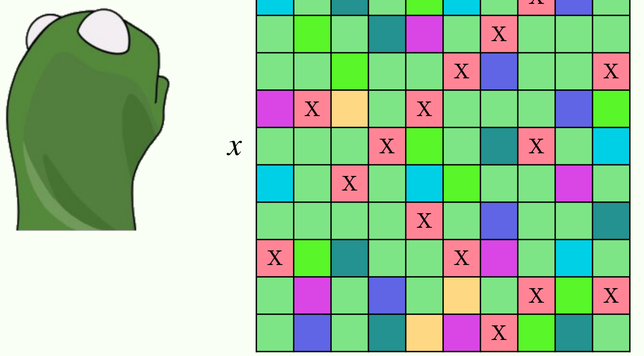

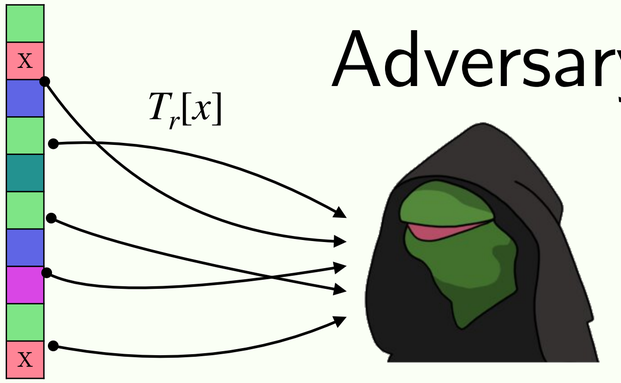

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See the construction

Faux-deterministic Construction for FIND1

Failure + Failure = Success ?

Faux-deterministic construction for FIND1

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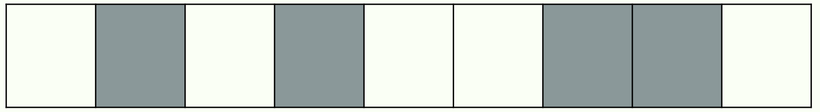

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$x =$

1	0	1	1	0	...	0	0	1

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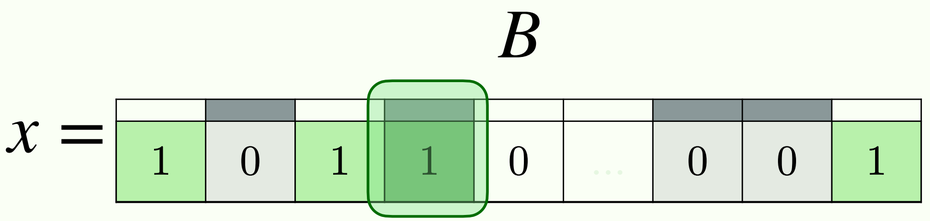

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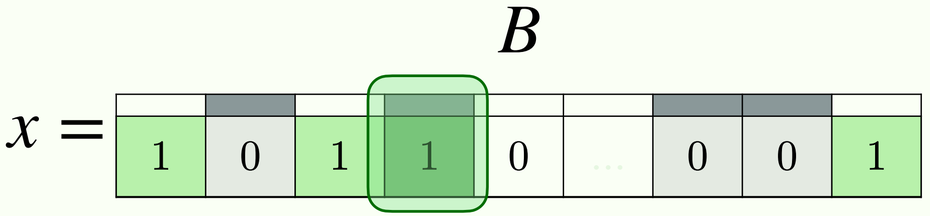

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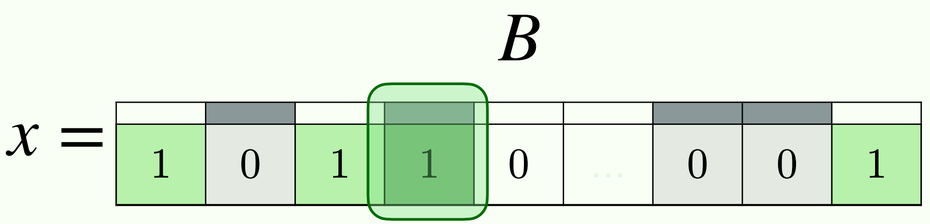

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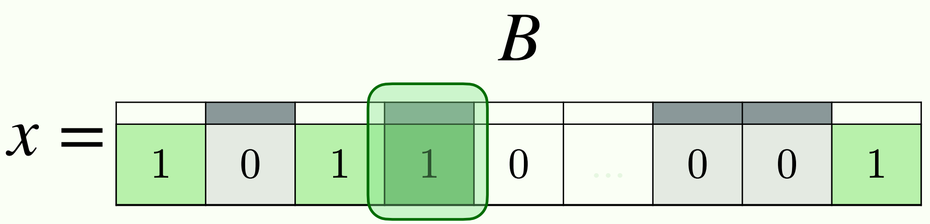

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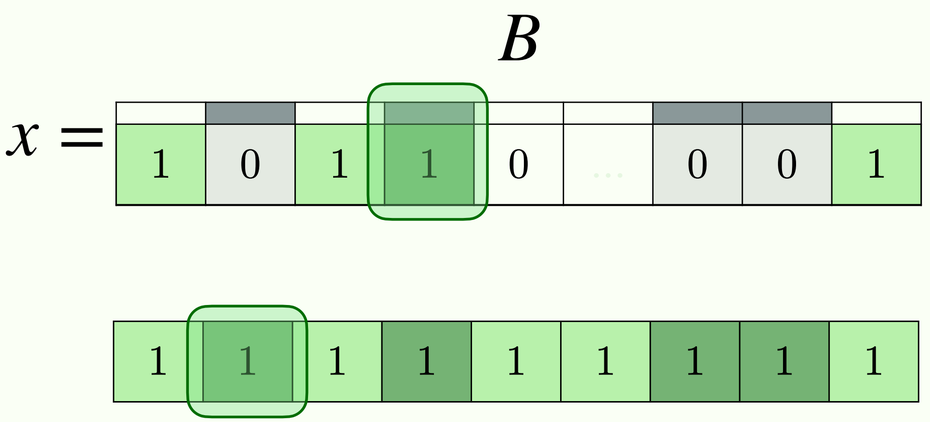

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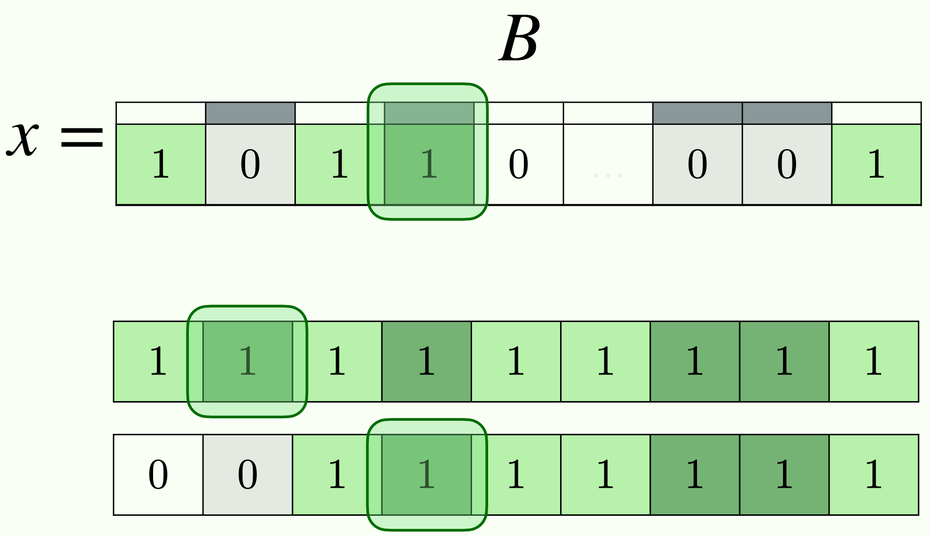

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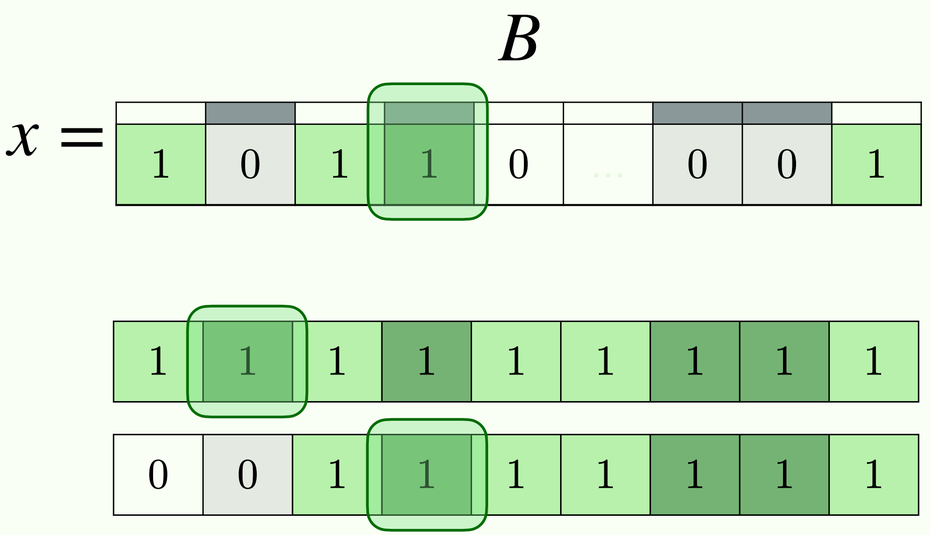

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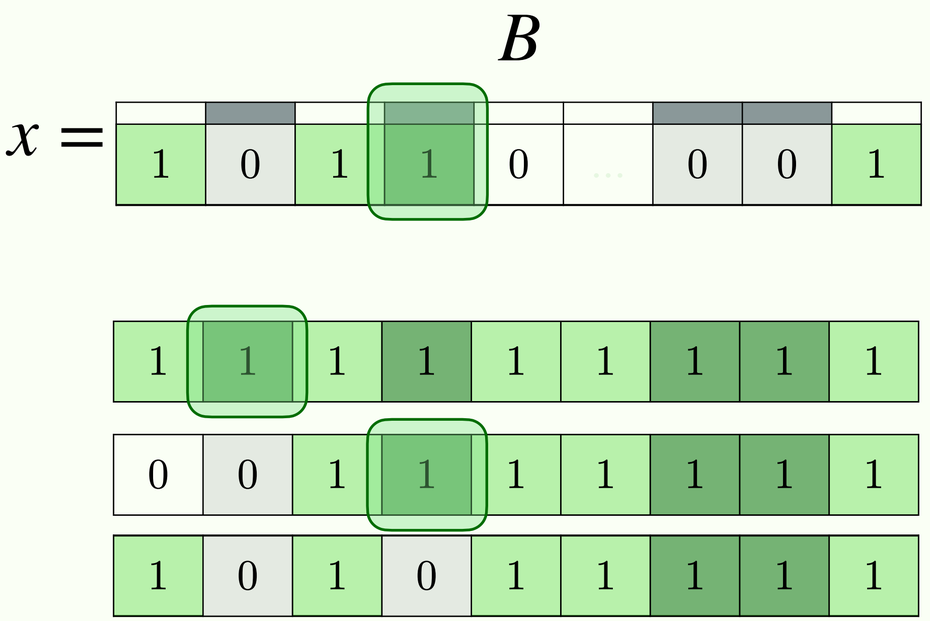

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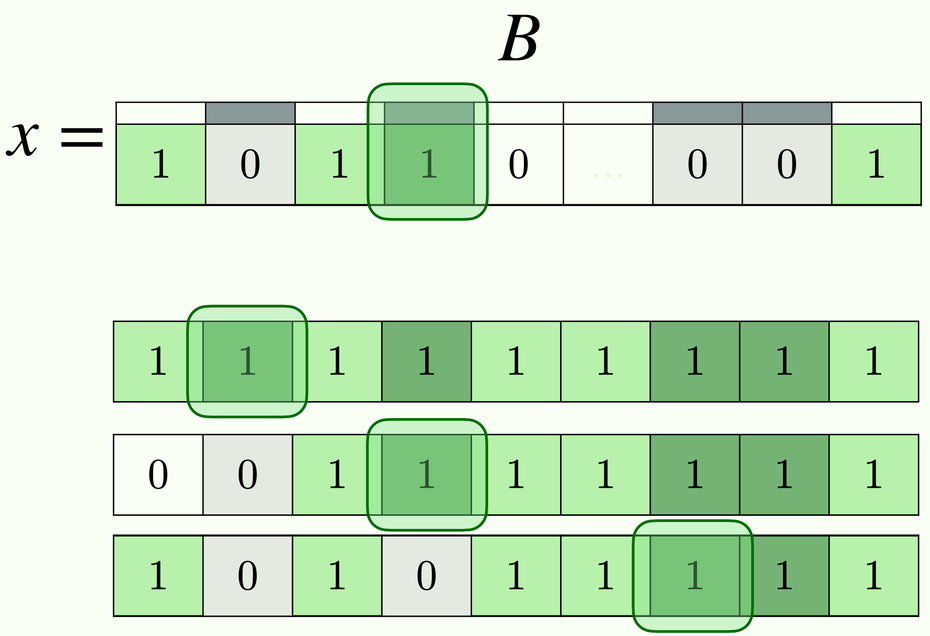

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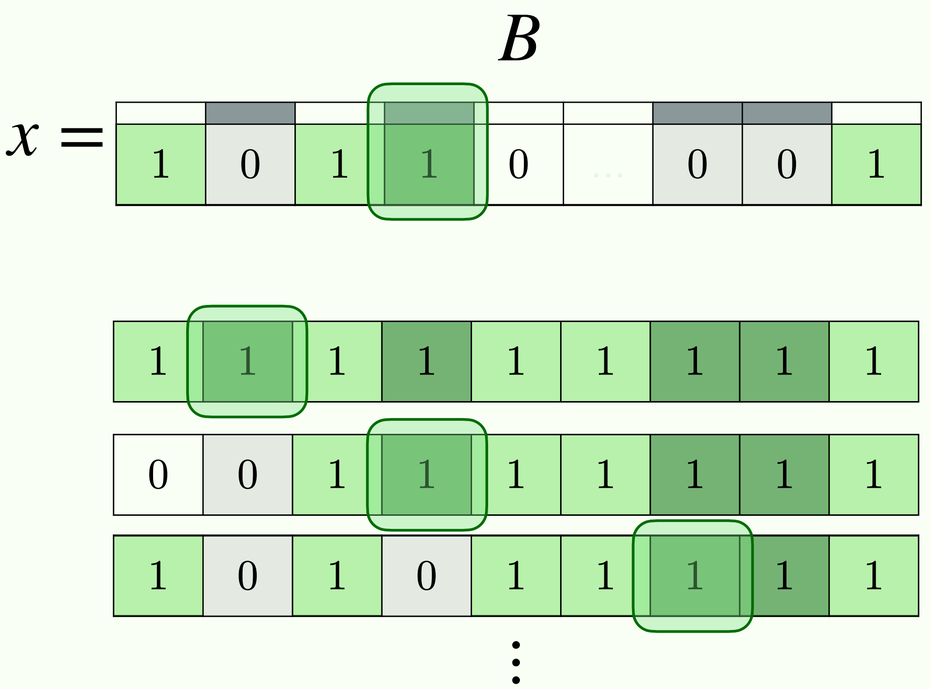

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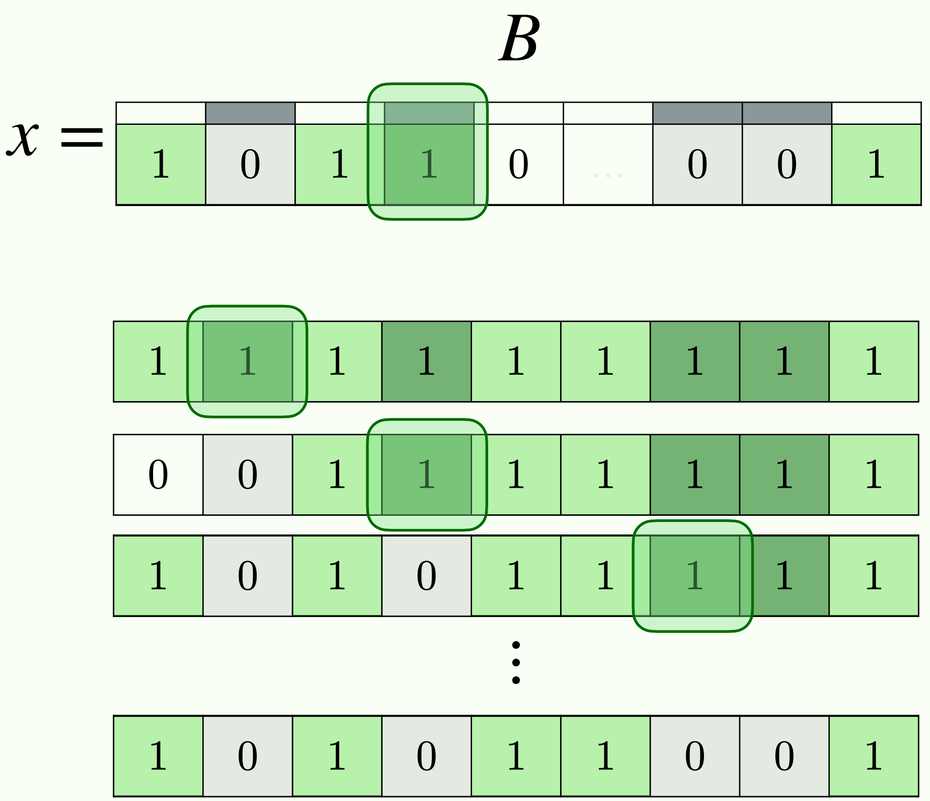

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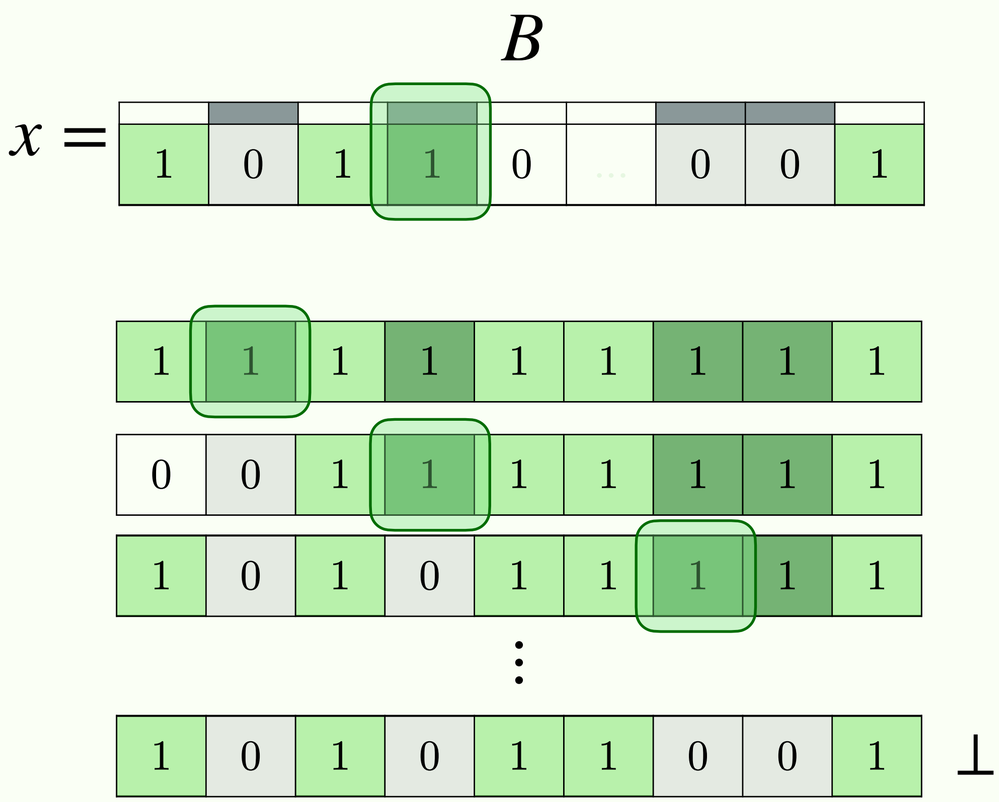

Faux-deterministic construction for FIND1

Attempt 2: adaptive approach

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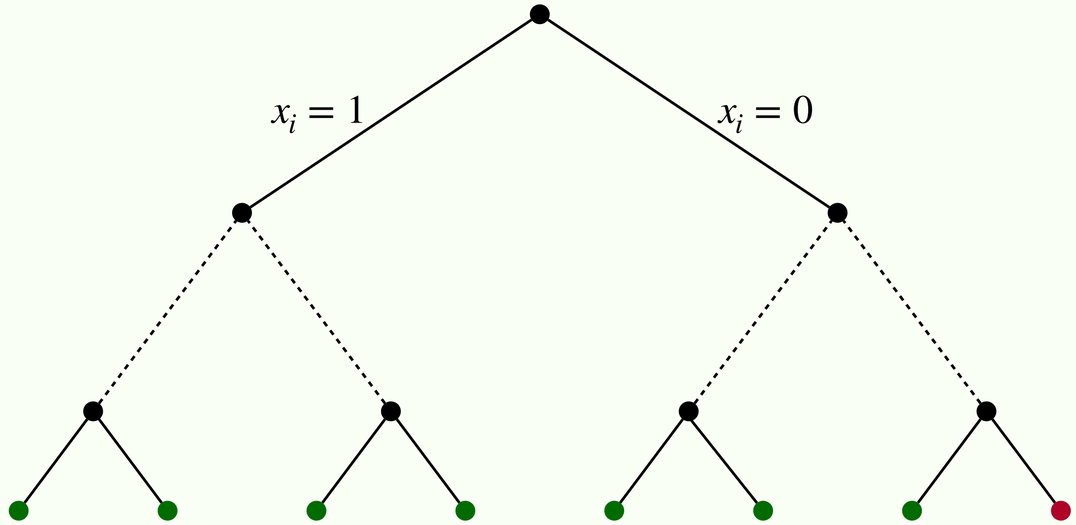

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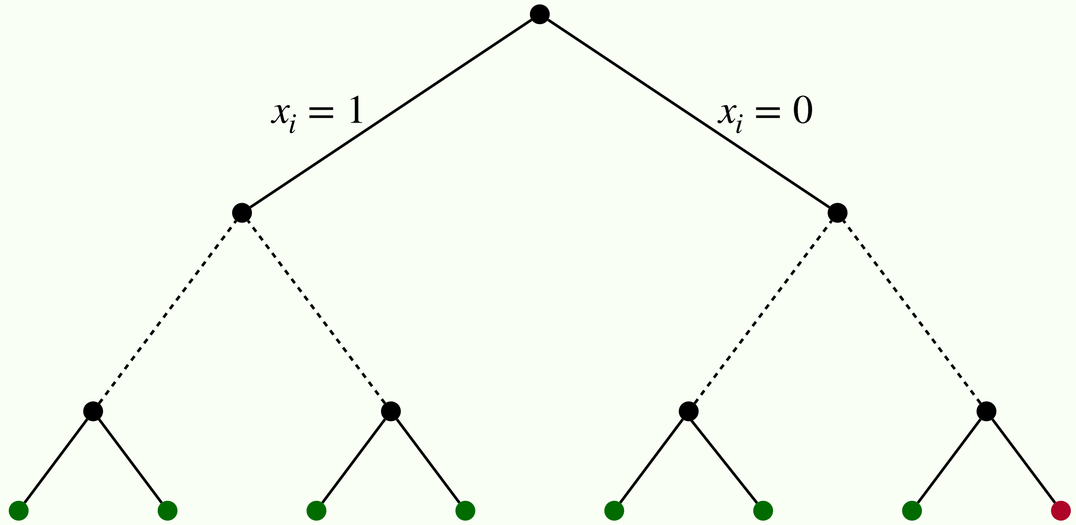
- run $T[x]$ to reach leaf $\langle \ell \rangle$



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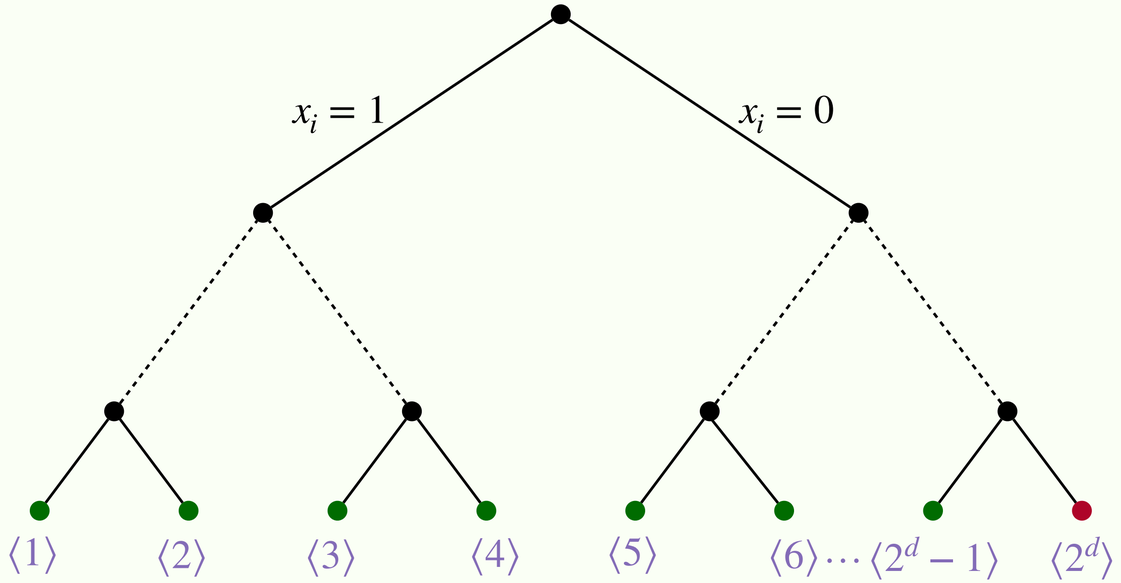

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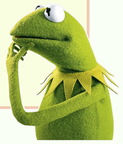

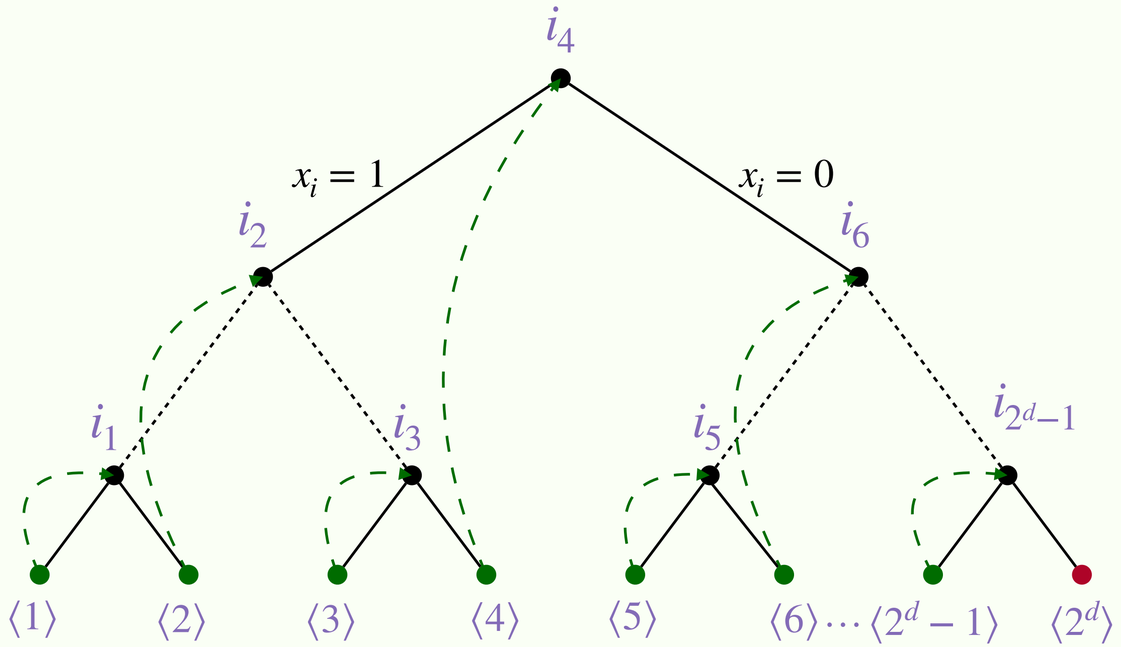

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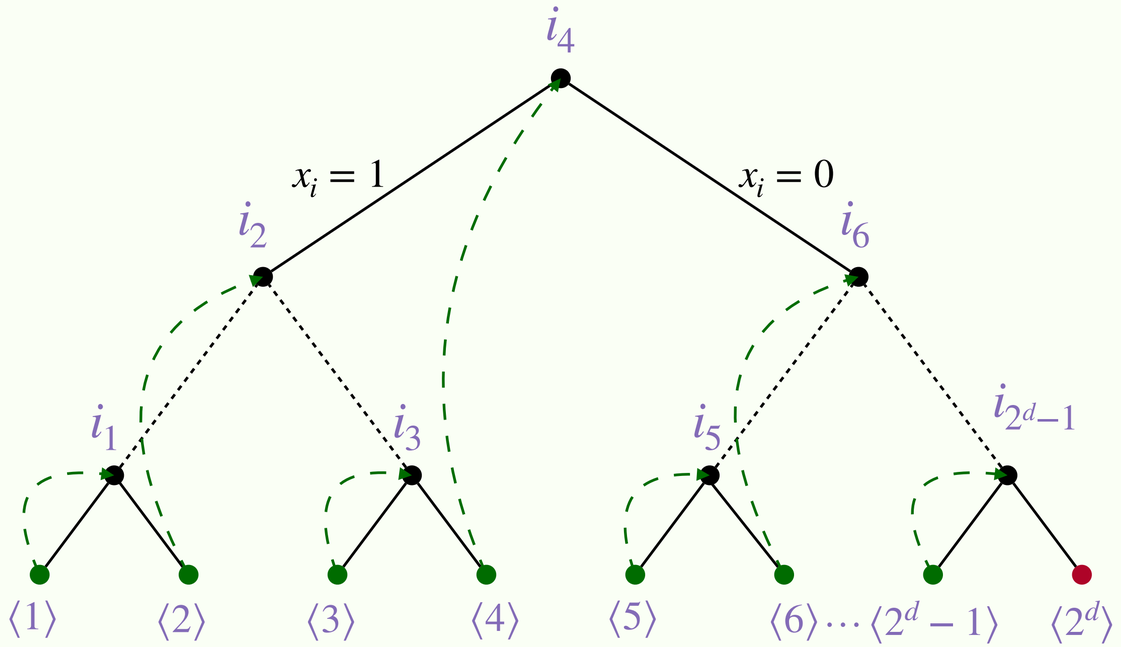

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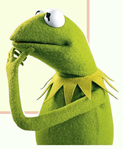

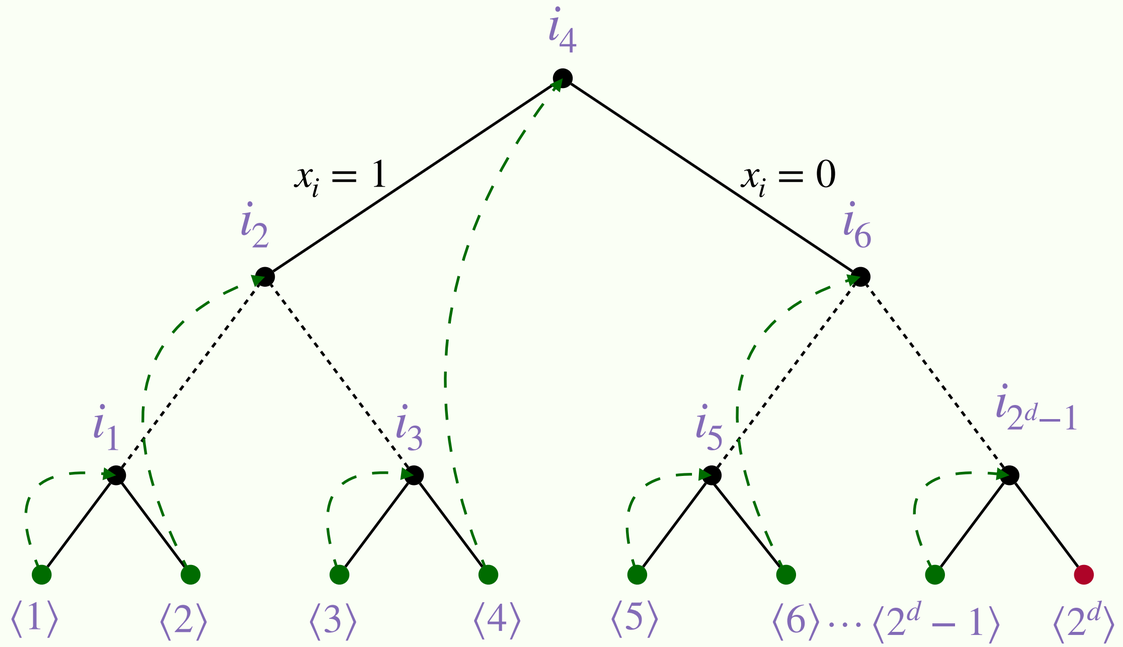

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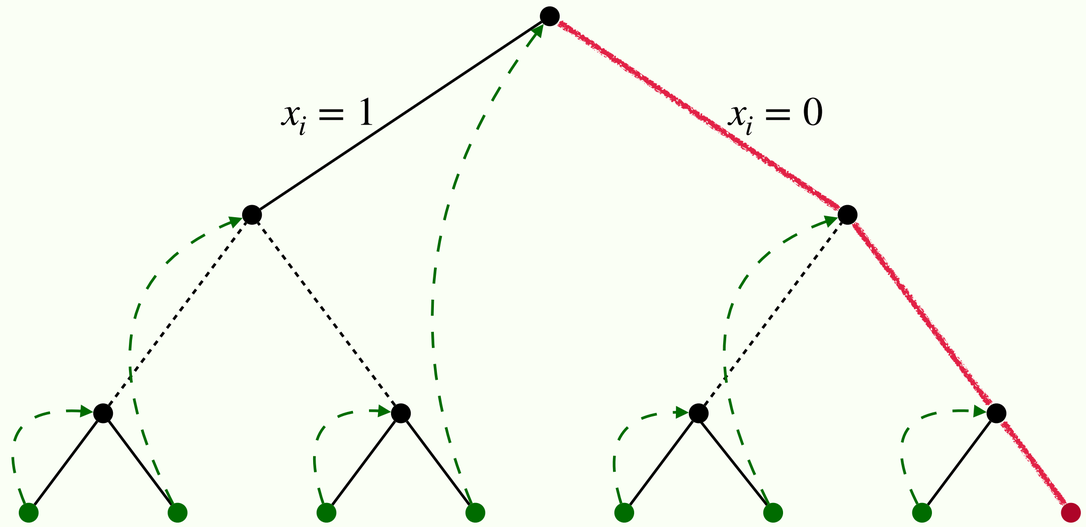

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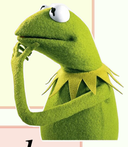

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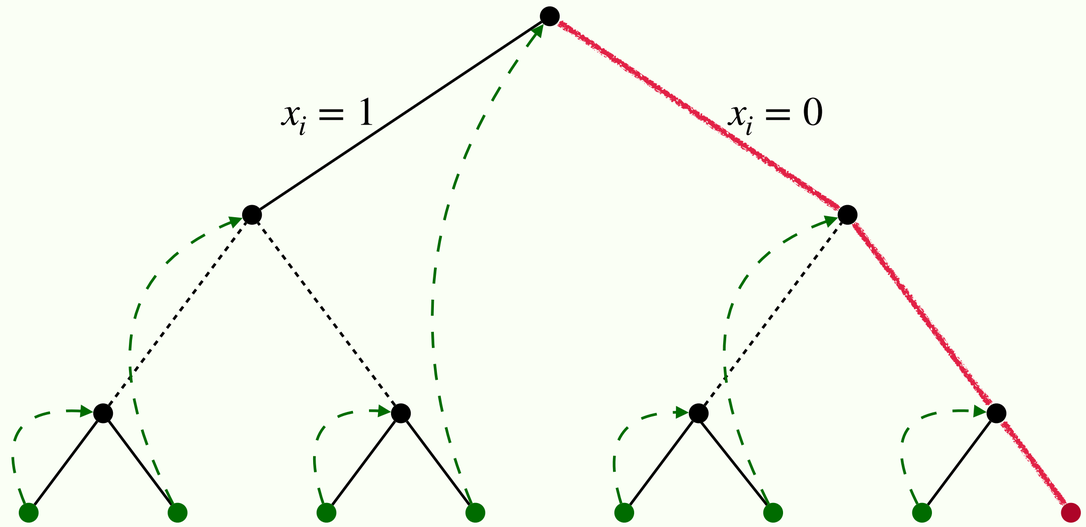

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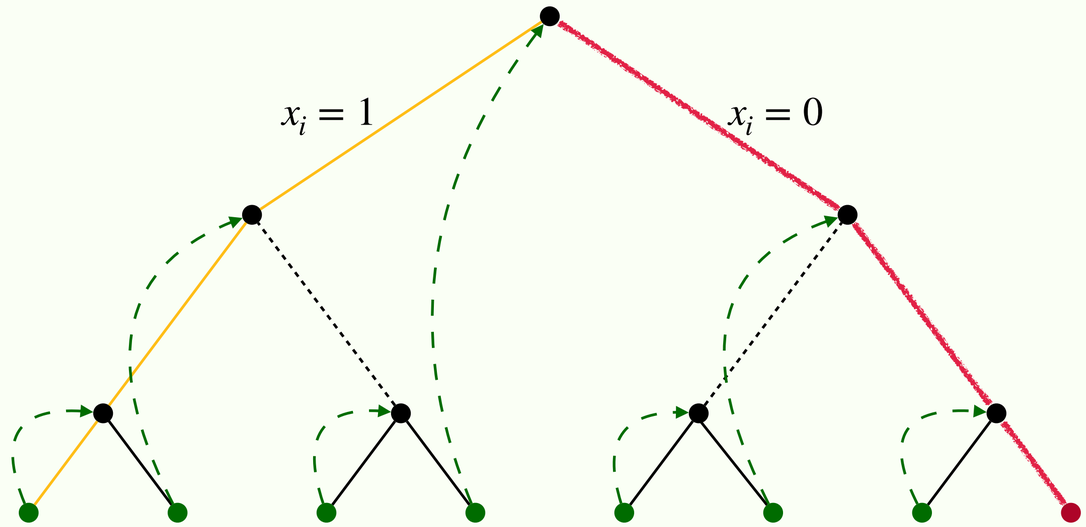

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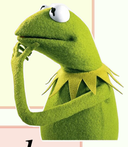

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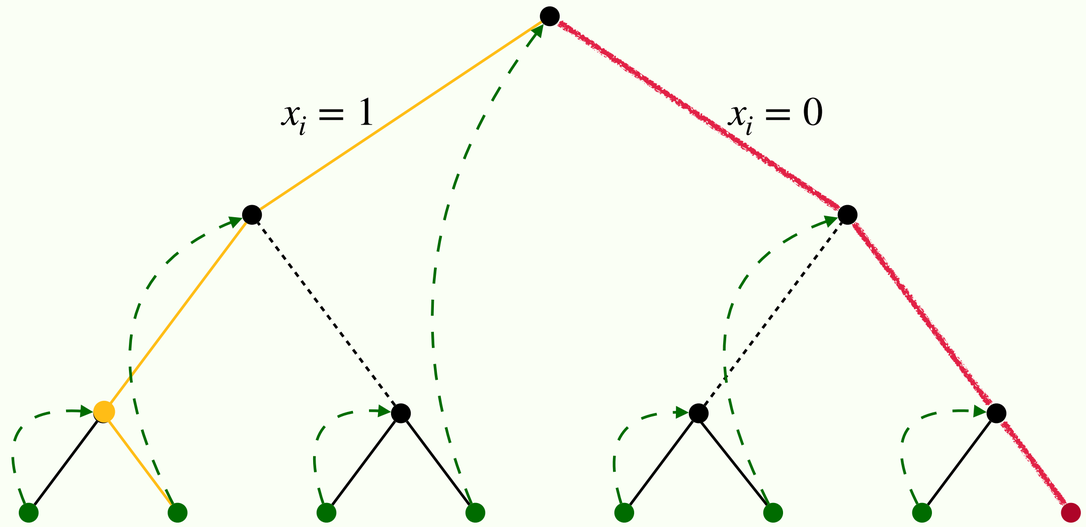

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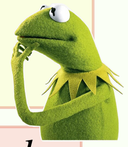

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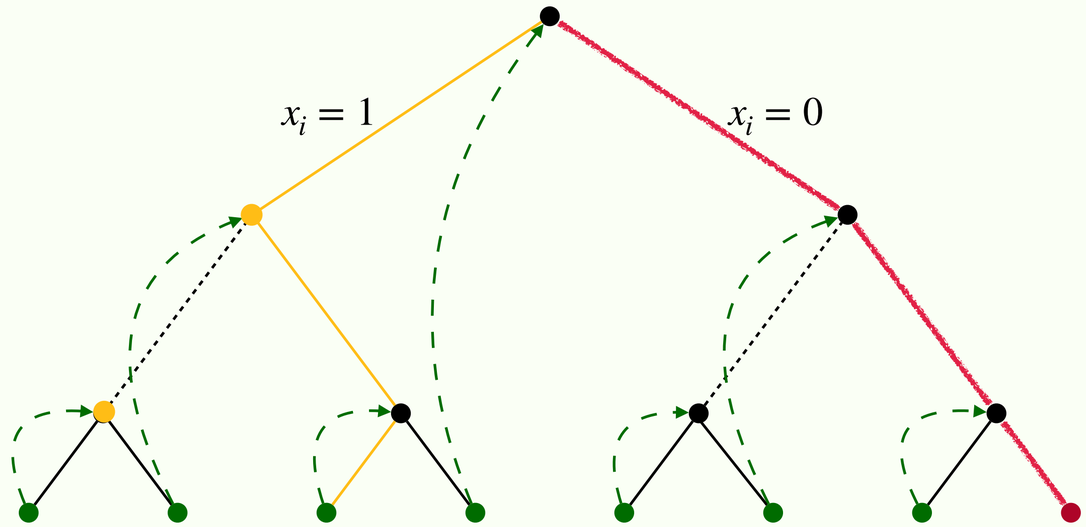

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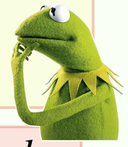

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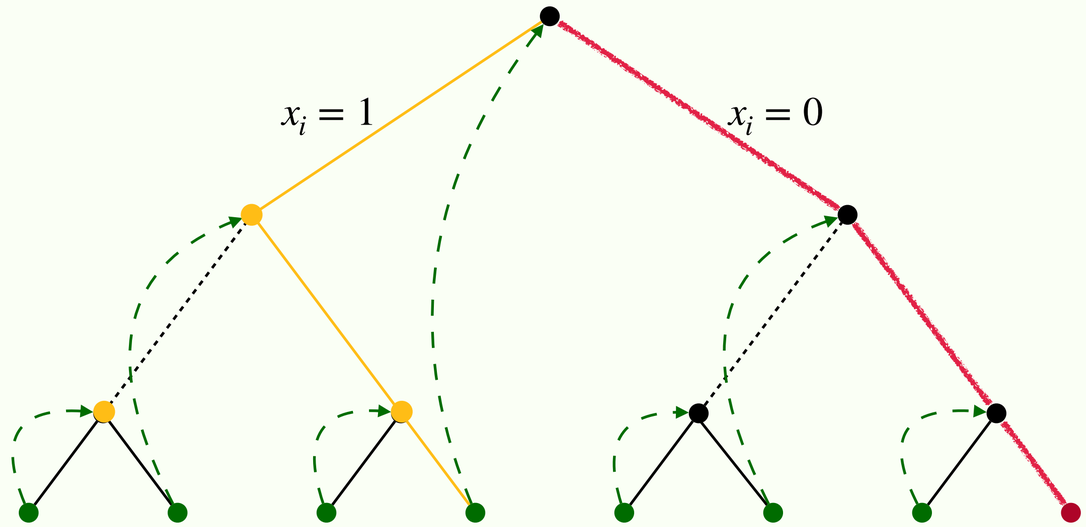

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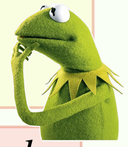

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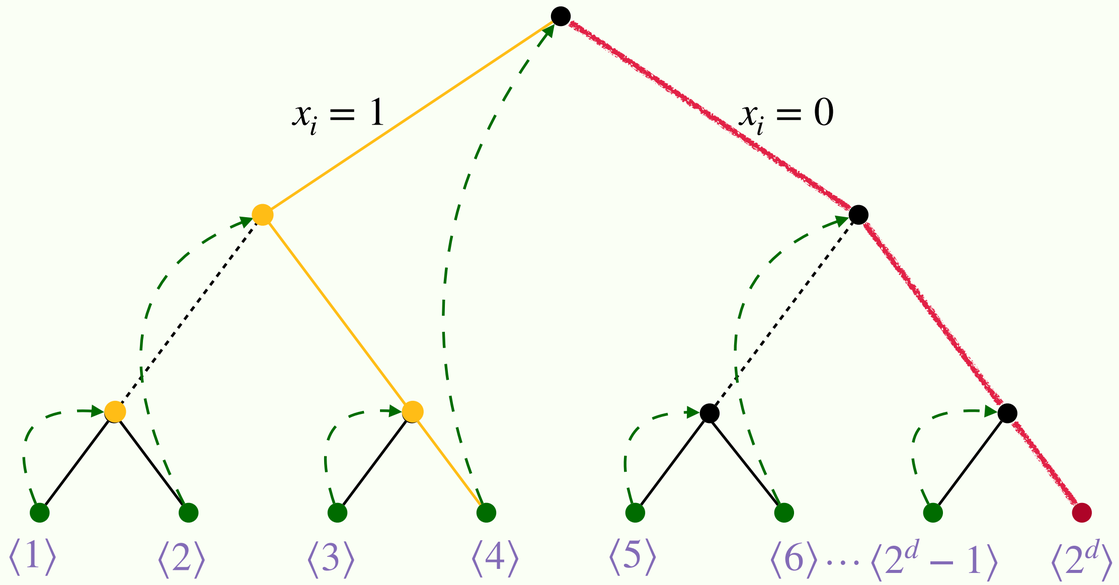

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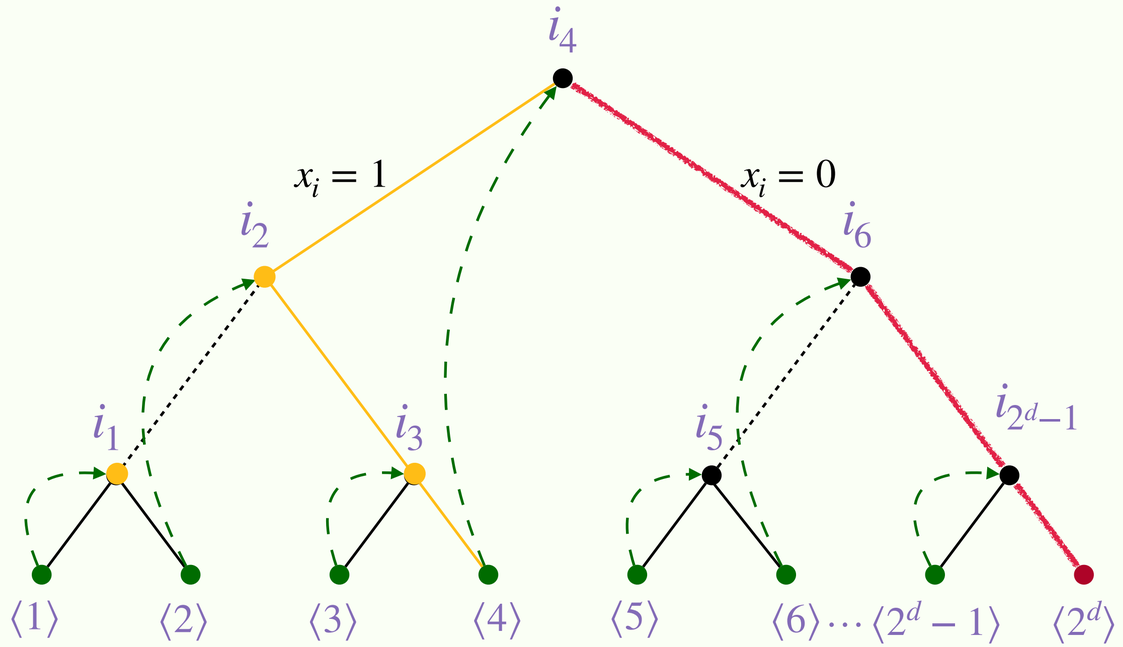

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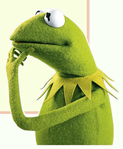

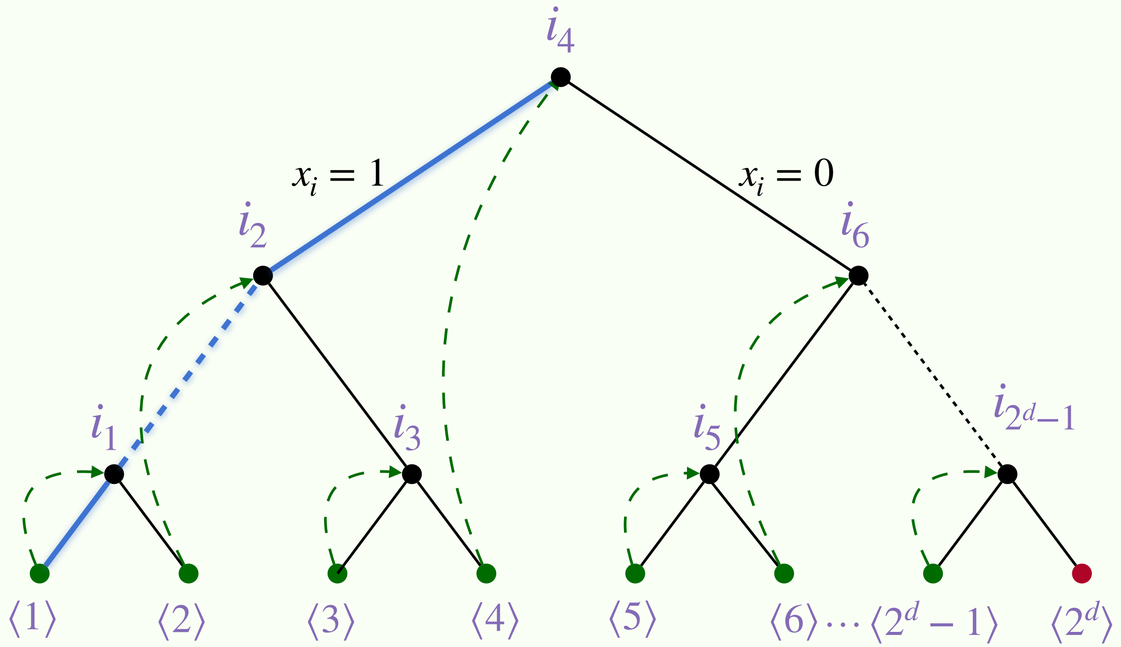

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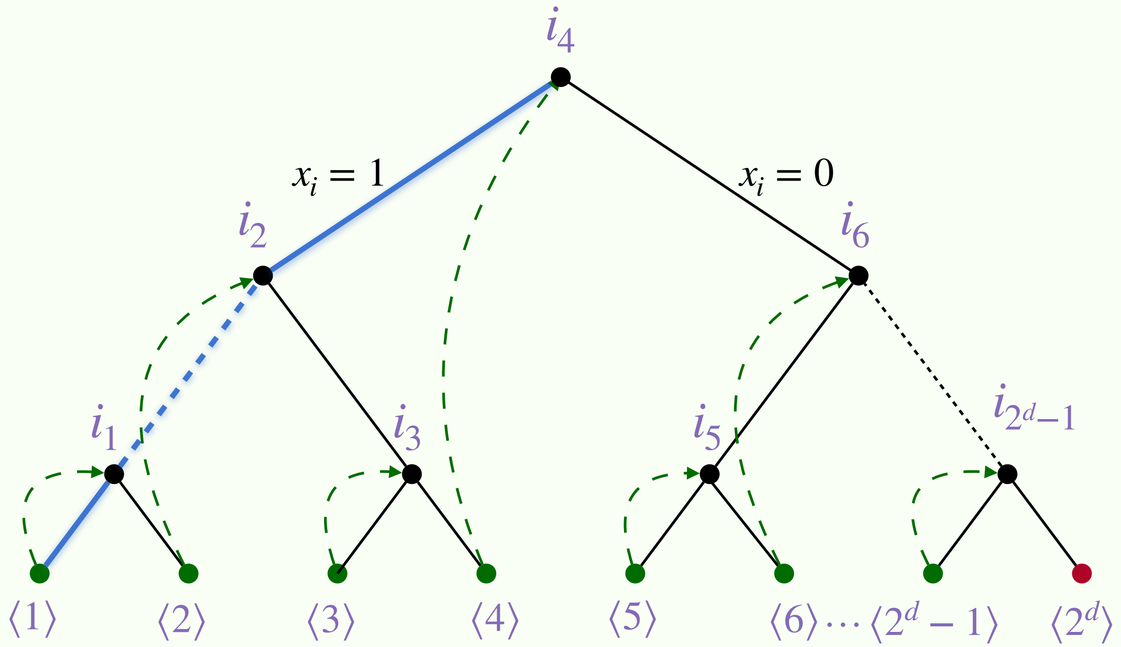

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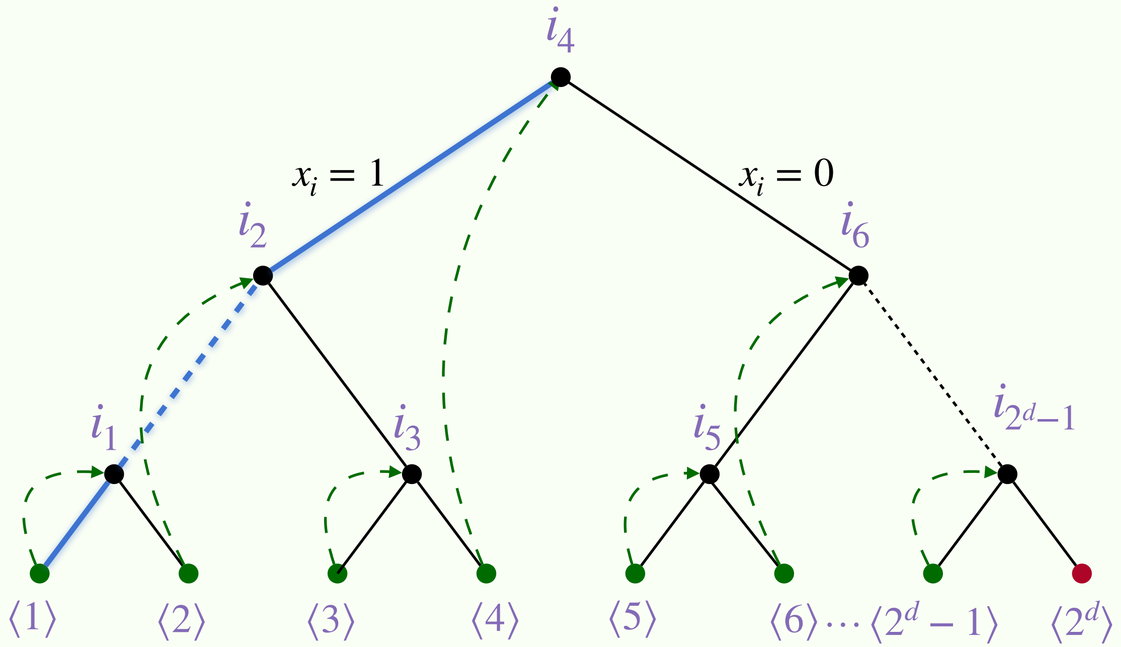

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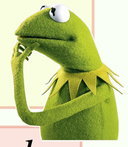

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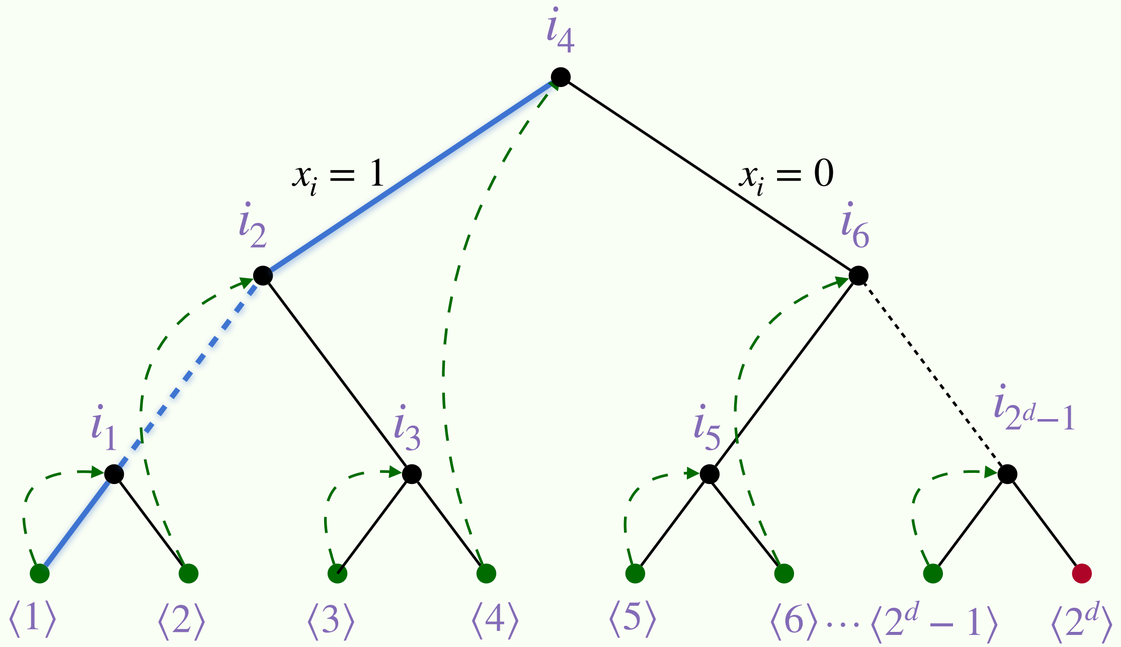

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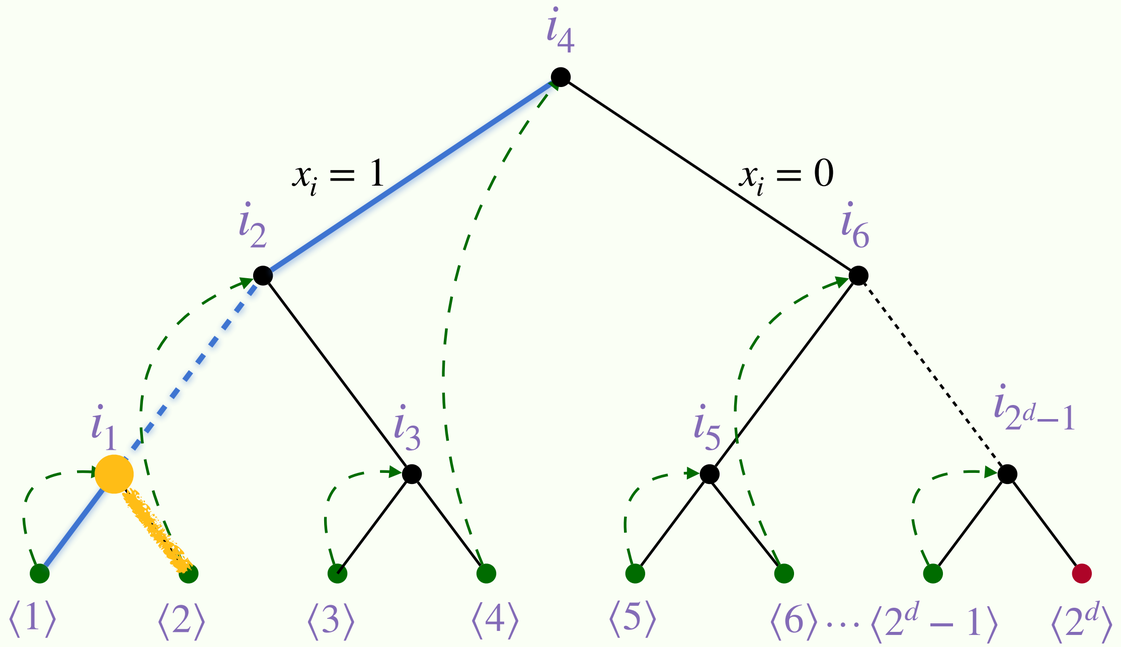

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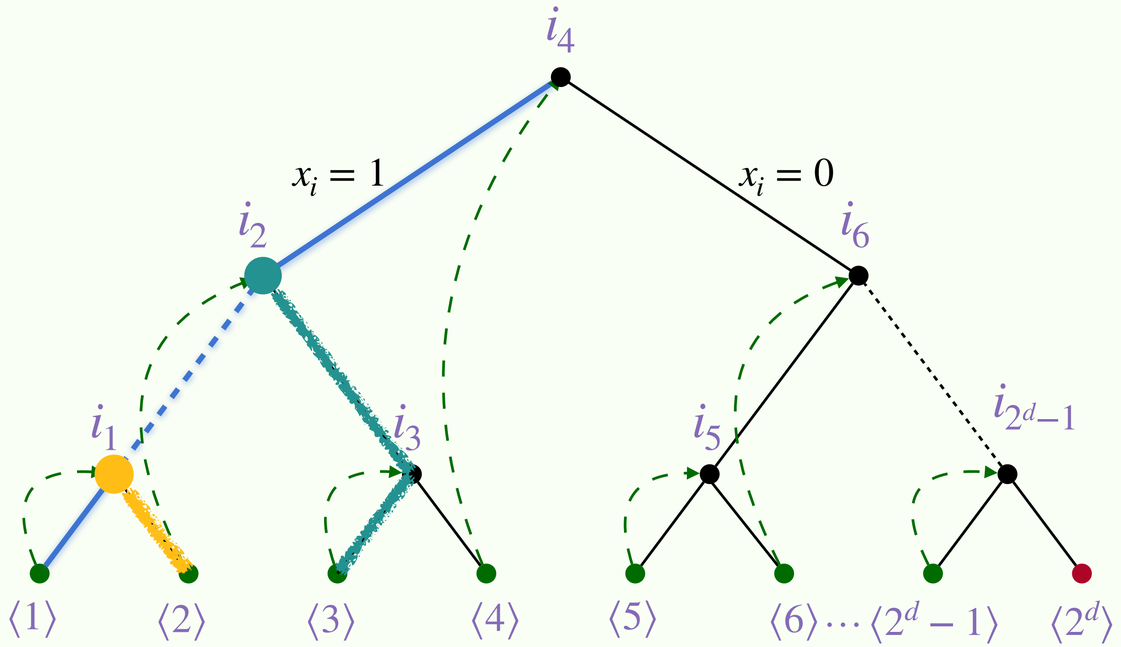

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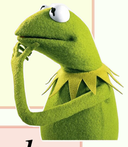

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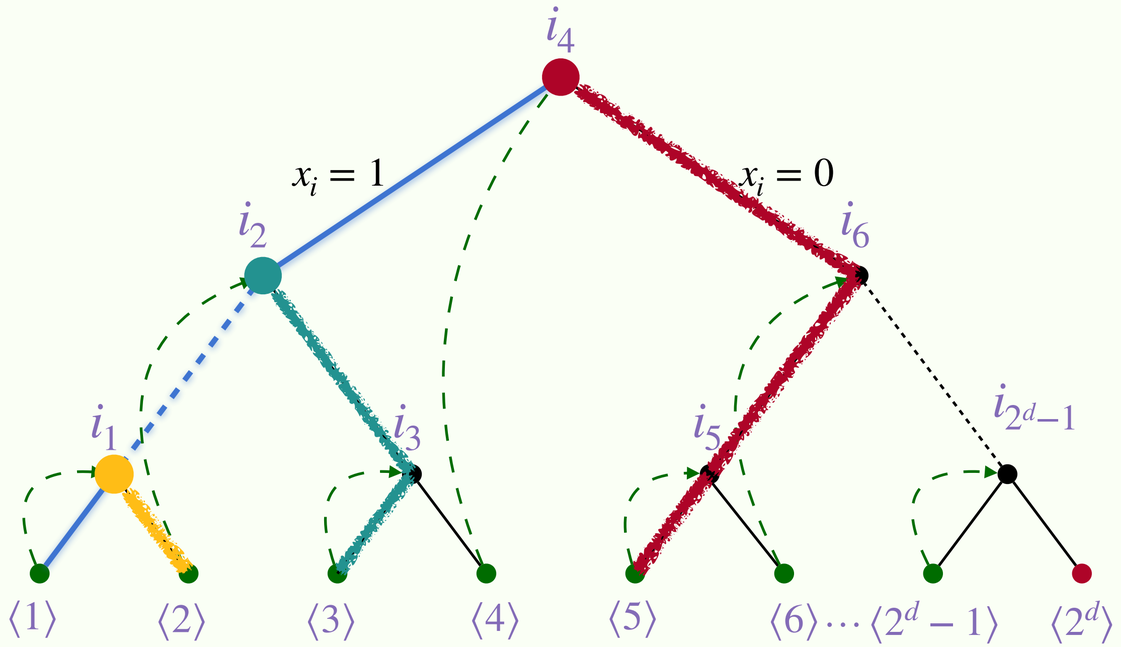

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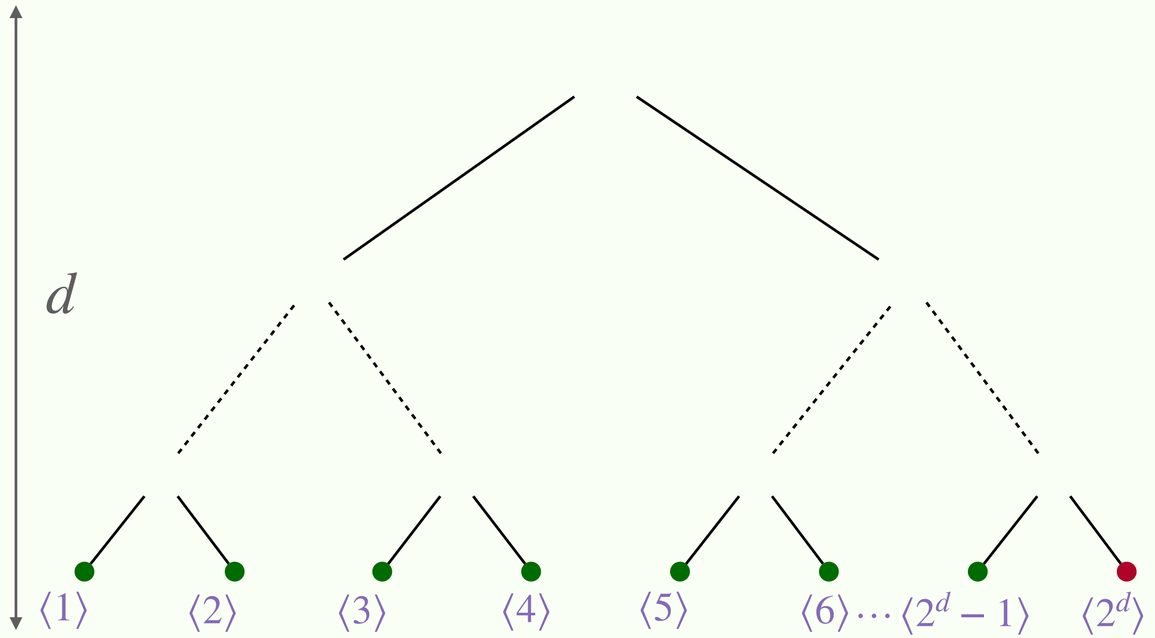

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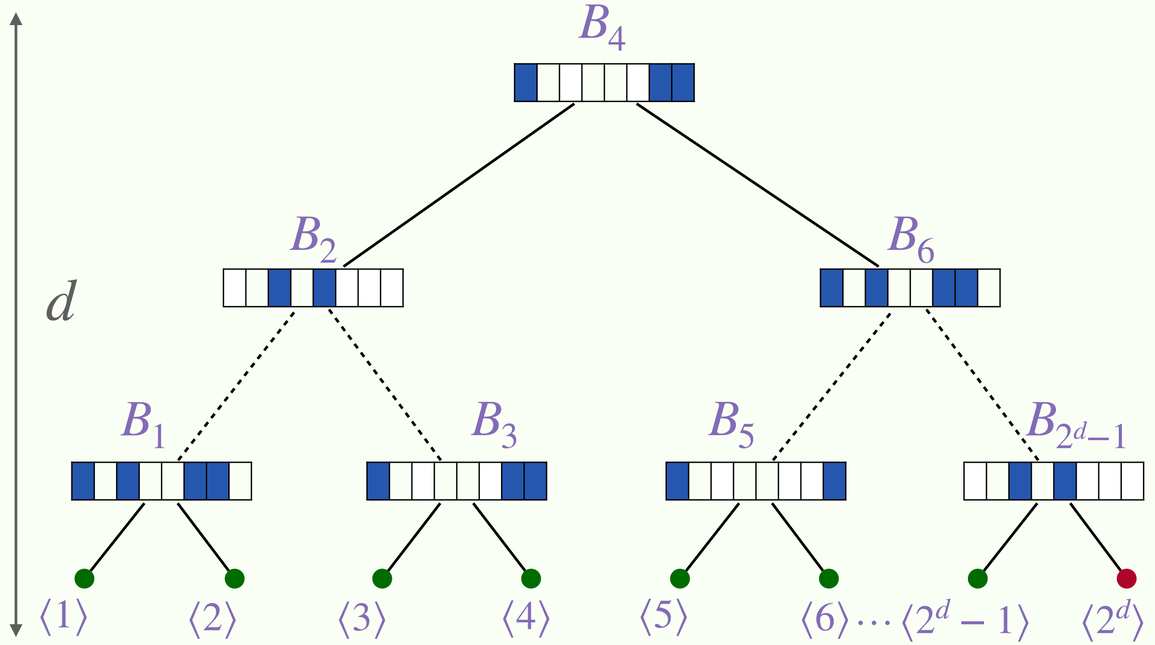

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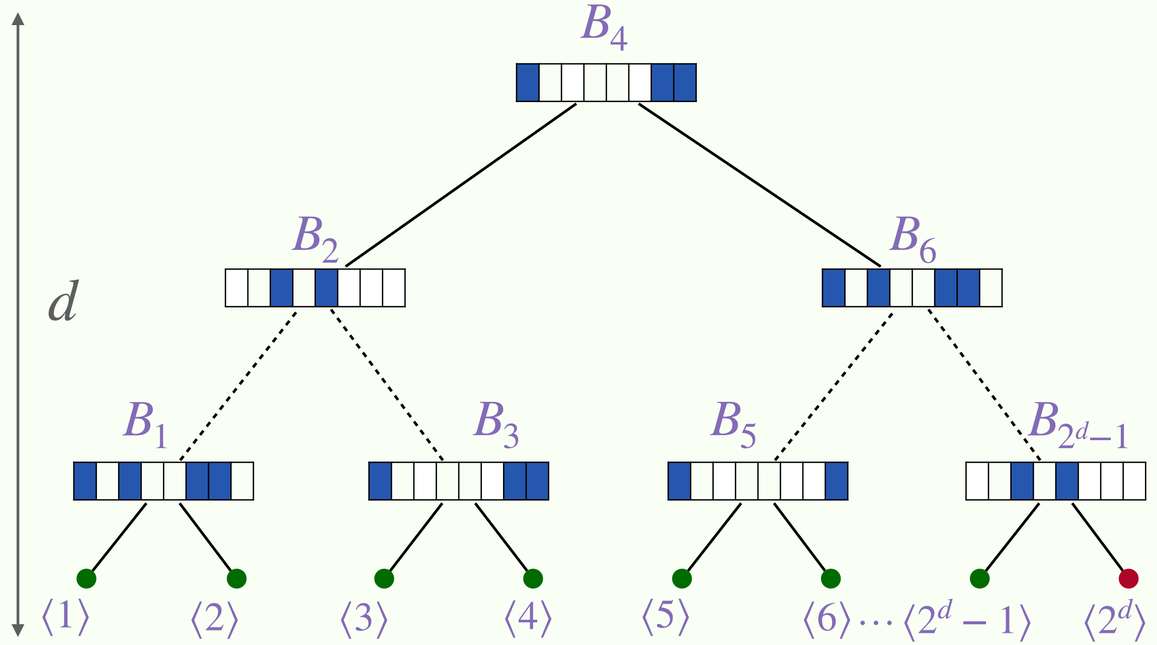

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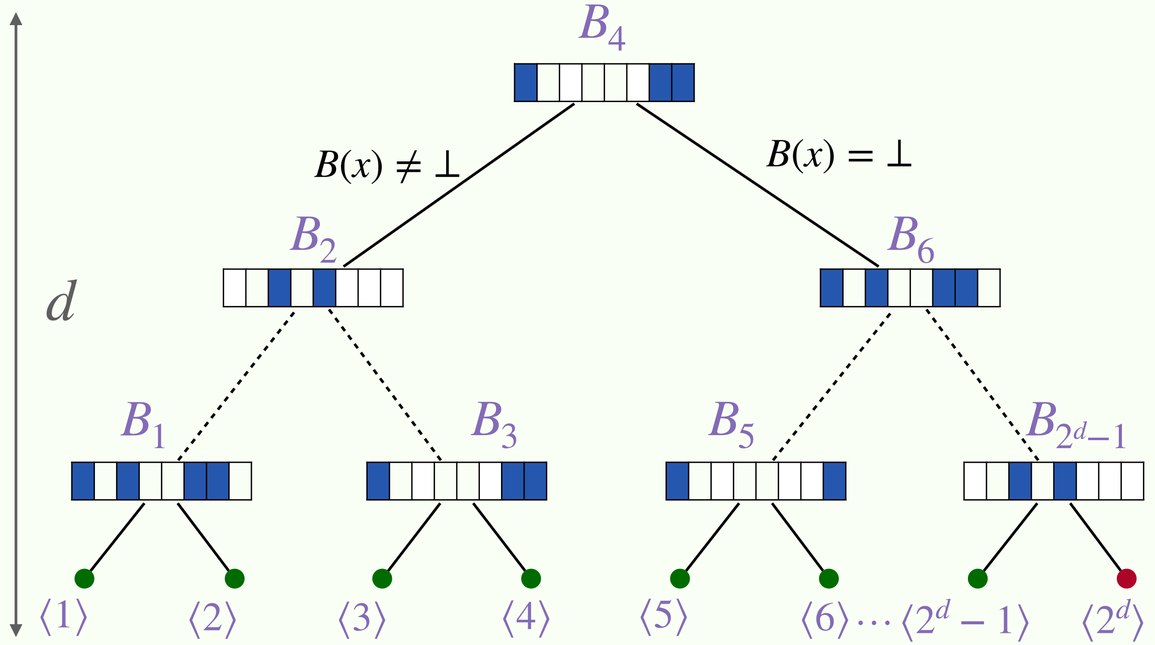

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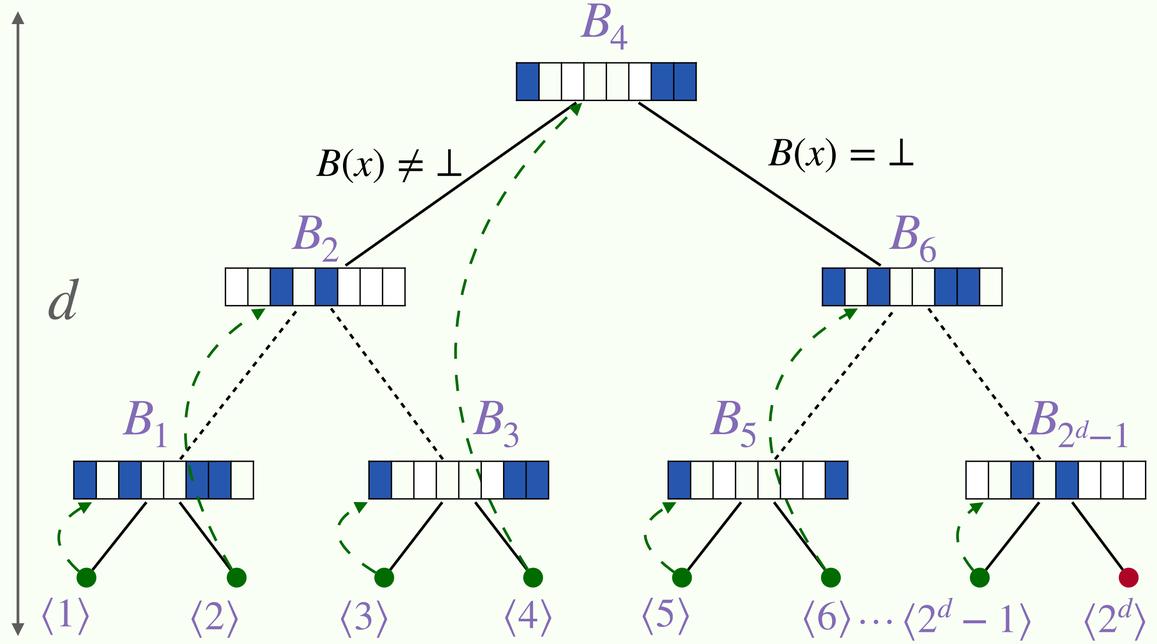

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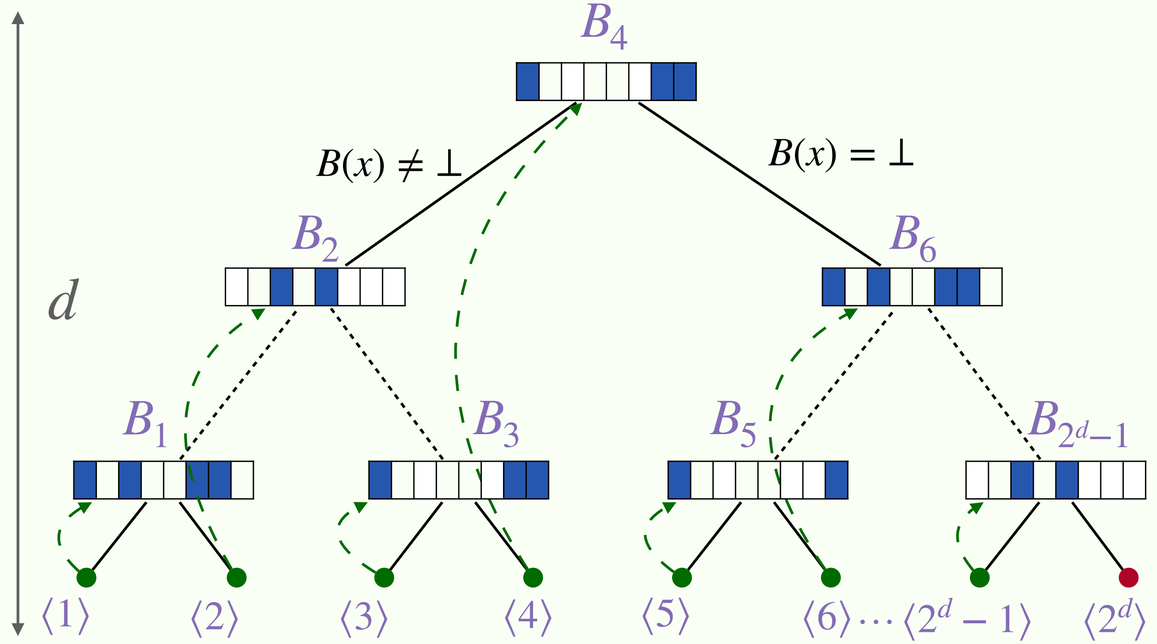

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Proof idea. Cannot skip any of the blocks, as earlier queries are hidden in the $\vee$ blocks

$\forall x, \Pr[x \text{ reaches } \langle \ell \rangle \mid B_1, \ldots, B_{\ell-1}] \geq (1 - 2^{-w})^d$

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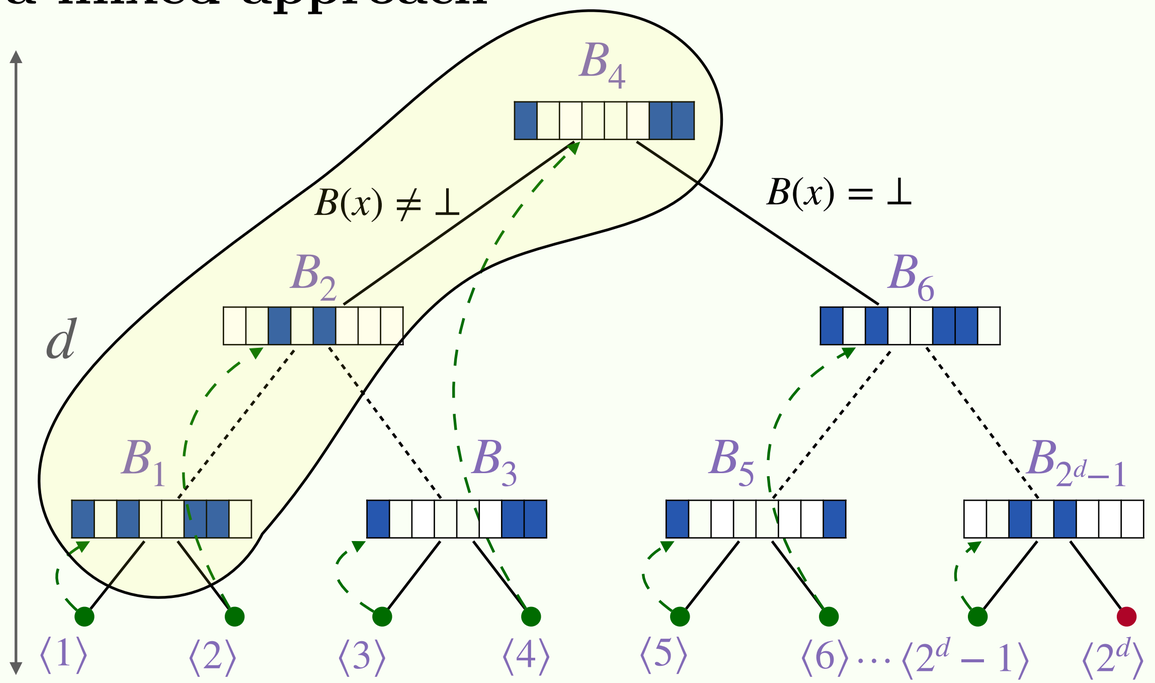

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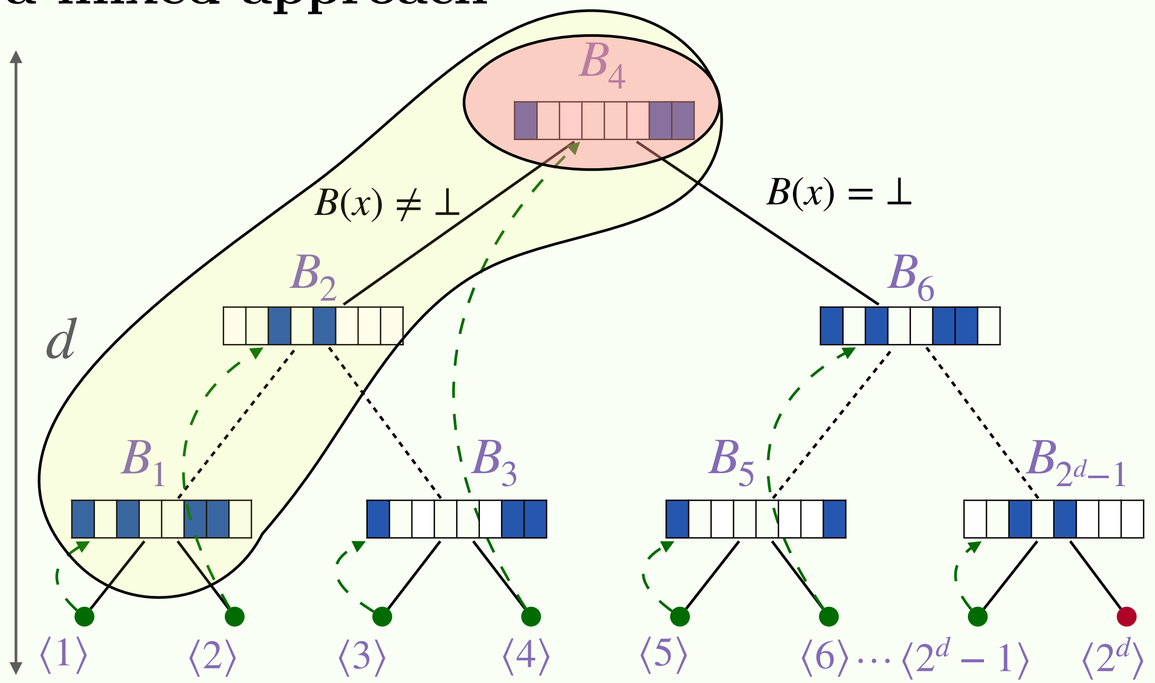

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The successful design: a mixed approach

Let $q = \text{polylog}(n)$, and $w, d := \Theta(\sqrt{q})$

- Sample $2^d - 1$ random blocks $B_v \subset [n]$, $|B_v| = w$
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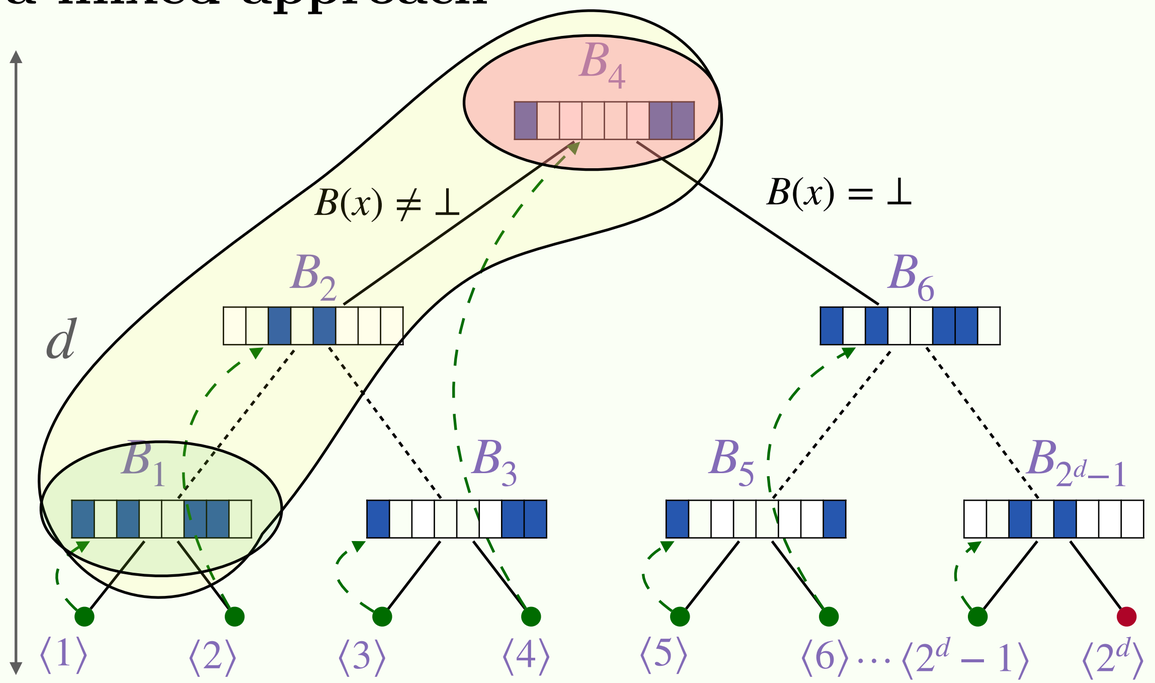

Claim. Let $q = \text{polylog}(n)$, and $w, d := \Theta(\sqrt{q})$. Then, any successful adversary requires at least $2^{\Omega(\sqrt{q})} = \text{quasipoly(n)}$ many queries.

Proof idea. Cannot skip any of the blocks, as earlier queries are hidden in the $\vee$ blocks

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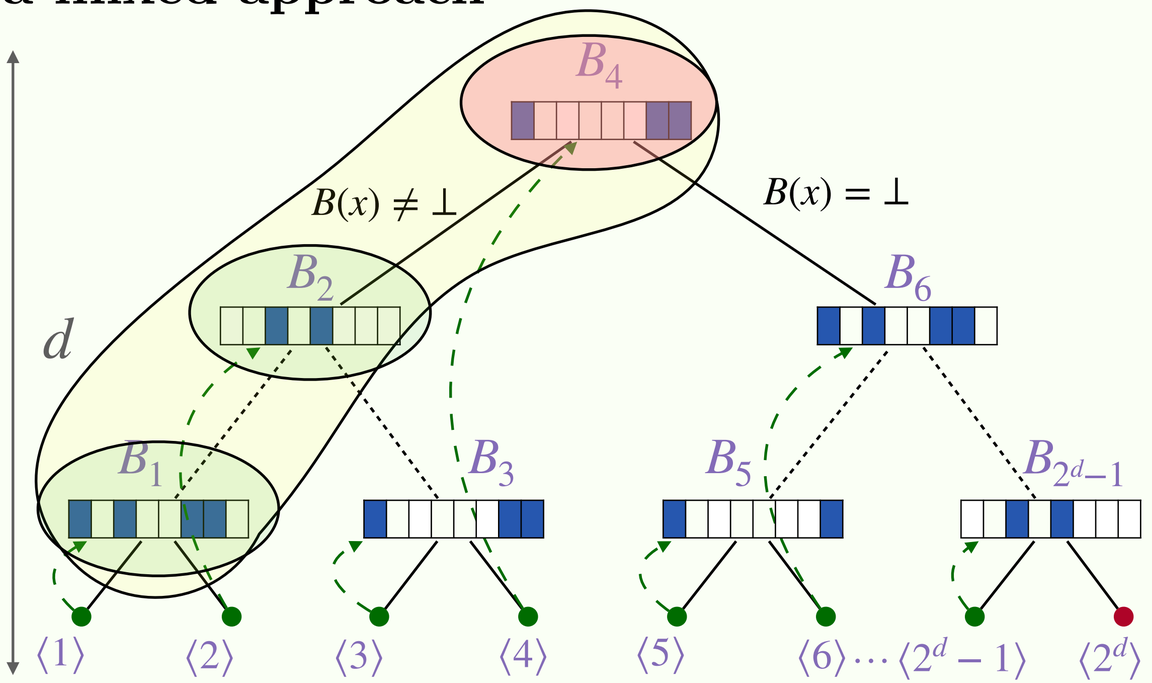

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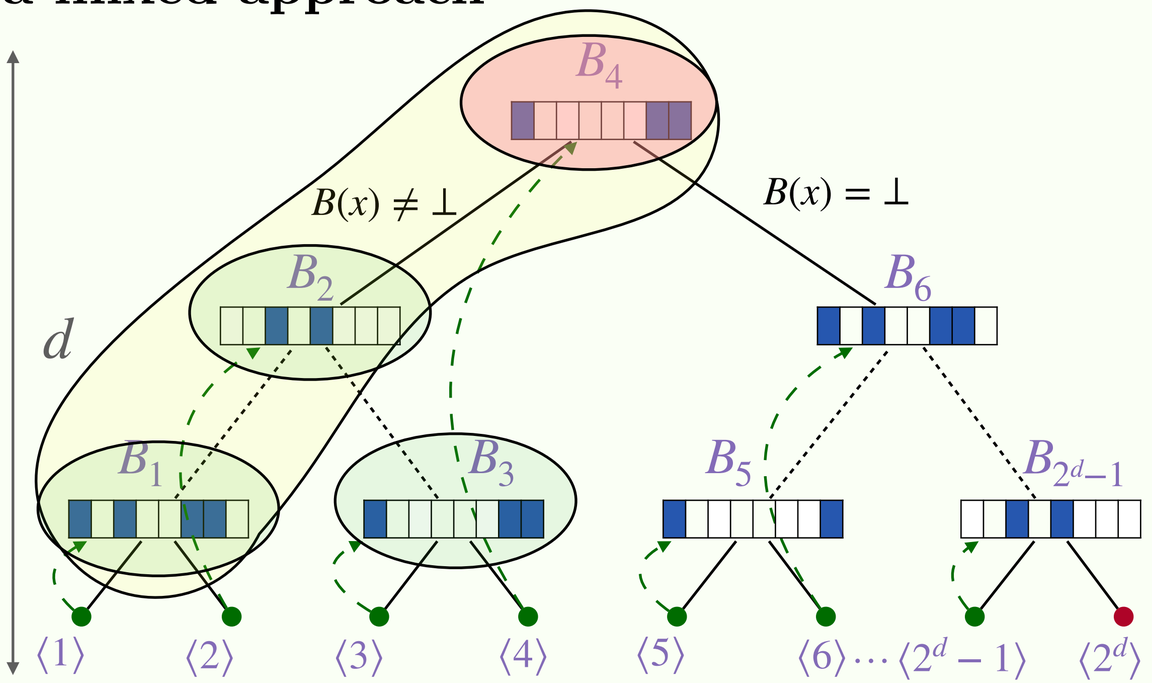

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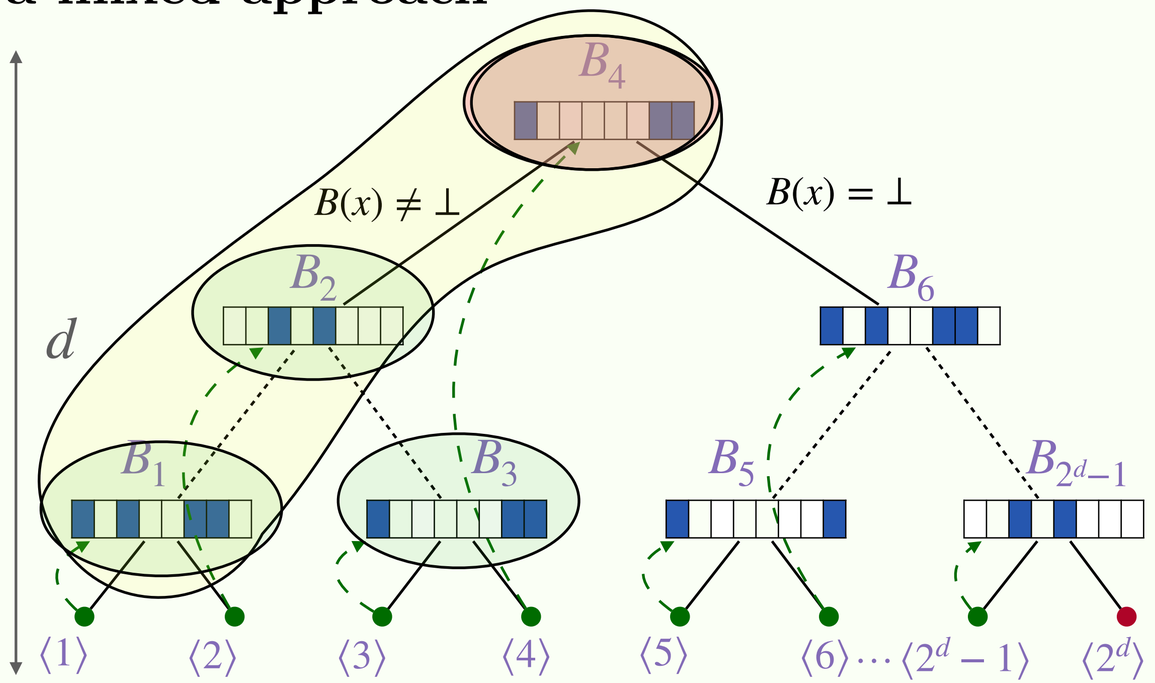

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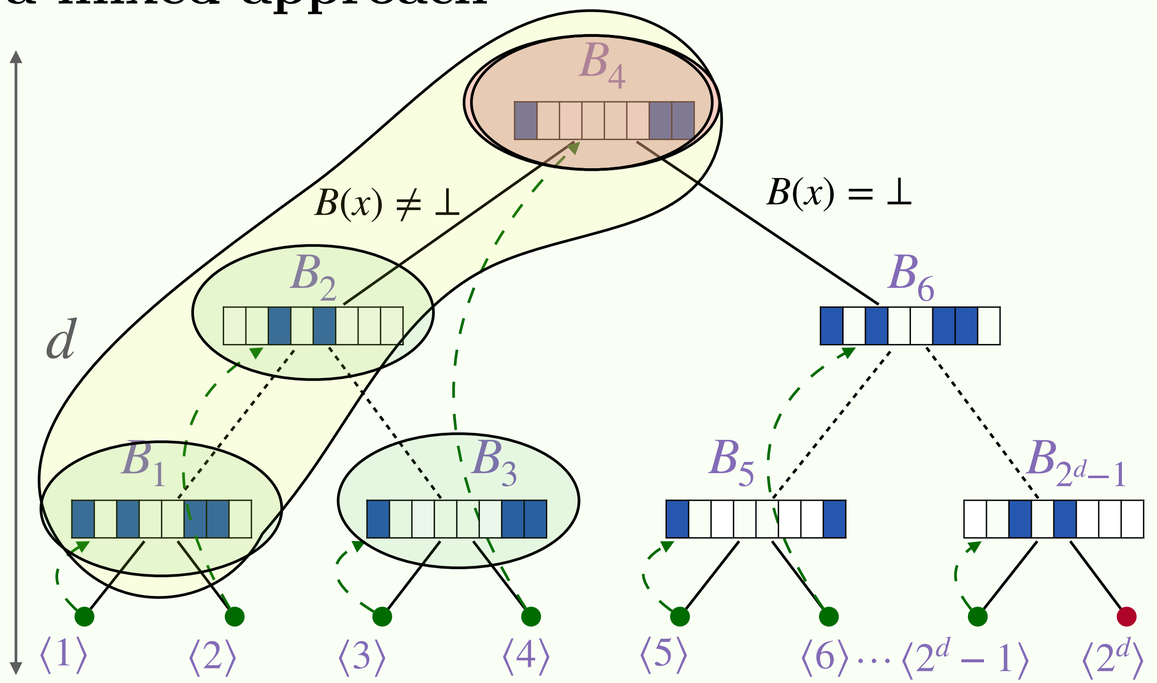

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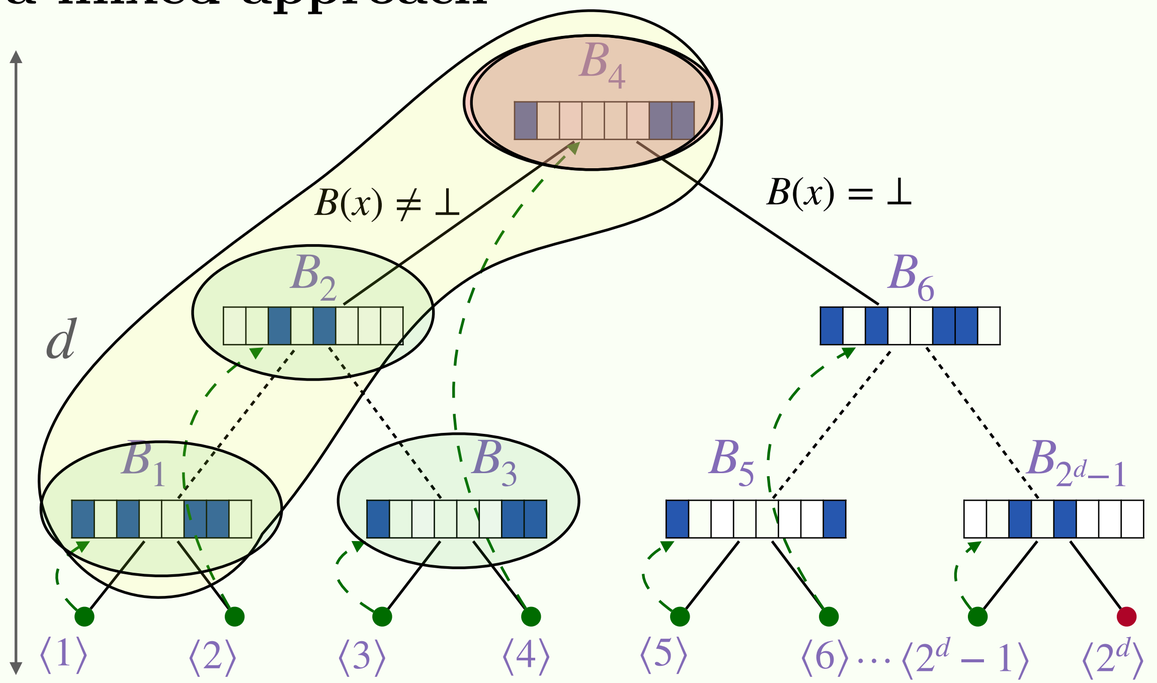

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Lemma. A similar bound of $2^{\Omega(\sqrt{q})}$ many queries can be proven against quantum adversaries.

Conclusion

In summary:

- Faux determinism: fix the randomness once; failures may exist, but bounded-query adversaries cannot find them.
- Faux derandomization: every efficiently verifiable randomized search problem has an efficient faux-deterministic algorithm
- Pseudo-deterministic lower bounds need to be non-constructive in a sense

Some future directions?

- Closing the gap $\sqrt{n} \lesssim D^{\text{pseudo}}(\text{FIND1}) \lesssim n$
- Constructing a faux-deterministic algorithm resisting $2^{\Omega(q)}$ many queries?
- In which models does faux determinists algorithms make sense
- Relation to other complexity measures?

Thank you!

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