A Tight Lower Bound for Entropy Flattening

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Agenda

1. Problem Definition / Model
2. Cryptographic Motivations
3. Proof Techniques

Flatness

Definition (Entropies)

Let X be a distribution over $\{0,1\}^n$. Define the surprise of x to be $H_X(x) = \log(1/\Pr[X = x])$.

$$H_{\mathsf{sh}}(X) \stackrel{\text{def}}{=} \mathop{\mathrm{E}}_{x \sim X}[H_X(x)],$$

$$H_{\mathsf{min}}(X) \stackrel{\text{def}}{=} \min_x H_X(x),$$

$$H_{\mathsf{max}}(X) \stackrel{\text{def}}{=} \log|\operatorname{Supp} X| \leq \max_x H_X(x).$$

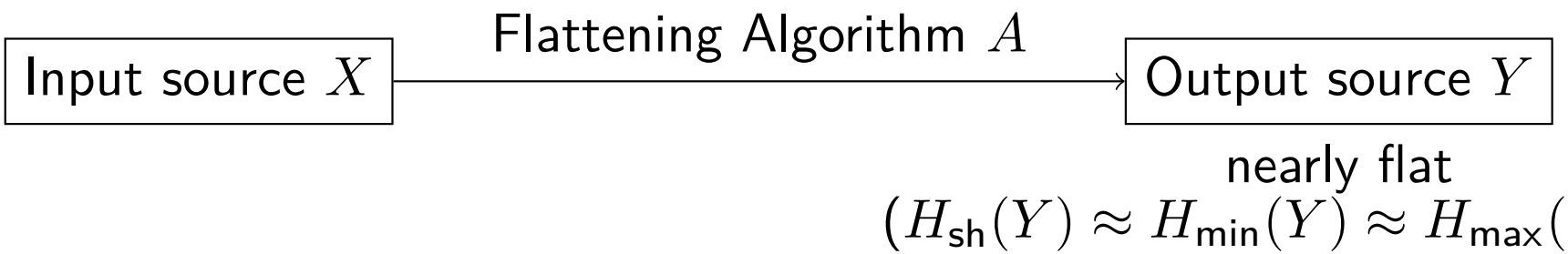
- $H_{\mathsf{min}}(X) \leq H_{\mathsf{sh}}(x) \leq H_{\mathsf{max}}(X)$ (The gap can be $\Theta(n)$.)
- A source X is flat iff $H_{\mathsf{sh}}(X) = H_{\mathsf{min}}(X) = H_{\mathsf{max}}(X)$.

Entropy Flattening

nearly flat

$(H_{\mathsf{sh}}(Y) \approx H_{\mathsf{min}}(Y) \approx H_{\mathsf{max}}(Y))$

Entropy Flattening



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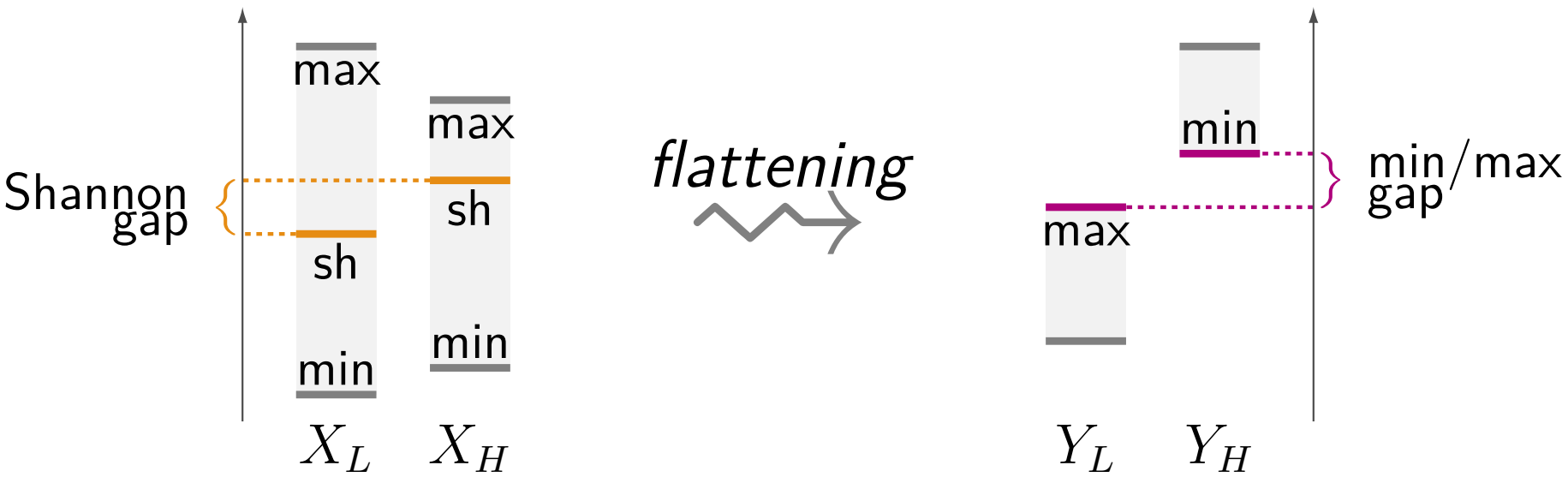

Entropy Flattening

Entropy Flattening Problem

Find an flattening algorithm A:

- If $H_{\mathsf{sh}}(X) \geq \tau + 1$, then $H^{\varepsilon}_{\mathsf{min}}(Y) \geq k + \Delta$.
- If $H_{\mathsf{sh}}(X) \leq \tau - 1$, then $H^{\varepsilon}_{\mathsf{max}}(Y) \leq k - \Delta$.

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Smooth Entropies

- $H^{\varepsilon}_{\mathsf{min}}(Y) \geq k$ if $\exists Y'$ s.t. $H_{\mathsf{min}}(Y) \geq k$ and $d_{\mathrm{TV}}(Y, Y') \leq \varepsilon$.
- $H^{\varepsilon}_{\mathsf{max}}(Y) \leq k$ if $\exists Y'$ s.t. $H_{\mathsf{max}}(Y) \leq k$ and $d_{\mathrm{TV}}(Y, Y') \leq \varepsilon$.

Solution: Repetition

Theorem ([HILL99, HR11])

- X: a distribution over $\{0,1\}^n$.
- Let $Y = (X_1, \ldots, X_q)$ where X_is are i.i.d. copies of X.

$$H^{\varepsilon}_{\mathsf{min}}(Y), H^{\varepsilon}_{\mathsf{max}}(Y) \in H_{\mathsf{sh}}(Y) \pm O\left(n\sqrt{q \log(1/\varepsilon)}\right)$$
$$q \cdot \left(H_{\mathsf{sh}}(X) \pm O\left(n\sqrt{\frac{\log(1/\varepsilon)}{q}}\right)\right)$$

(Asymptotic Equipartition Property (AEP) in information theory)

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(Asymptotic Equipartition Property (AEP) in information theory)

- $q = O(n^2)$ is sufficient for the constant entropy gap.
- $q = \Omega(n^2)$ is needed due to anti-concentration results. [HR11]

Query Model

The Model:

- **Input source**: encoded by a function $f : \{0,1\}^n \to \{0,1\}^m$ and defined as $f(U_n)$.
- **Flattening algorithm**: oracle algorithm $A^f : \{0,1\}^{n'} \to \{0,1\}^{m'}$ has query access to f.
- **Output source**: $A^f(U_{n'})$.
- **Example**: $A^f(r_1, \ldots, r_q) = (f(r_1), \ldots, f(r_q))$

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More powerful:

- Querying correlated positions or even in an adaptive way.
- Computation on the query inputs. e.g., hashing

Main Theorems

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Flattening algorithms for n-bit oracles f require $\Omega(n^2)$ oracle queries.

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Def: SDU Algorithm

- $H_{\mathsf{sh}}(f(U_n)) \geq \tau + 1 \quad \Rightarrow \quad d_{\mathrm{TV}}(A^f(U_{n'}), U_{m'}) < \varepsilon.$
- $H_{\mathsf{sh}}(f(U_n)) \leq \tau - 1 \quad \Rightarrow \quad \mathrm{Supp}\left(A^f(U_{n'})\right)/2^{m'} \leq \varepsilon.$

Flattening Algorithm $\iff$ SDU Algorithm

(Reduction between two NISZK-complete problems [GSV99])

Theorem

SDU algorithms for n-bit oracles f require $\Omega(n^2)$ oracle queries.

Connection to Cryptographic Constructions

Example: OWF $f \to$ PRG g^f ([HILL90, Hol06, HHR06, HRV10, VZ13]):

1. Create a gap between "pseudoentropy" and (true) entropy.
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Can we do better in the entropy flattening step?

Overview of the Proof

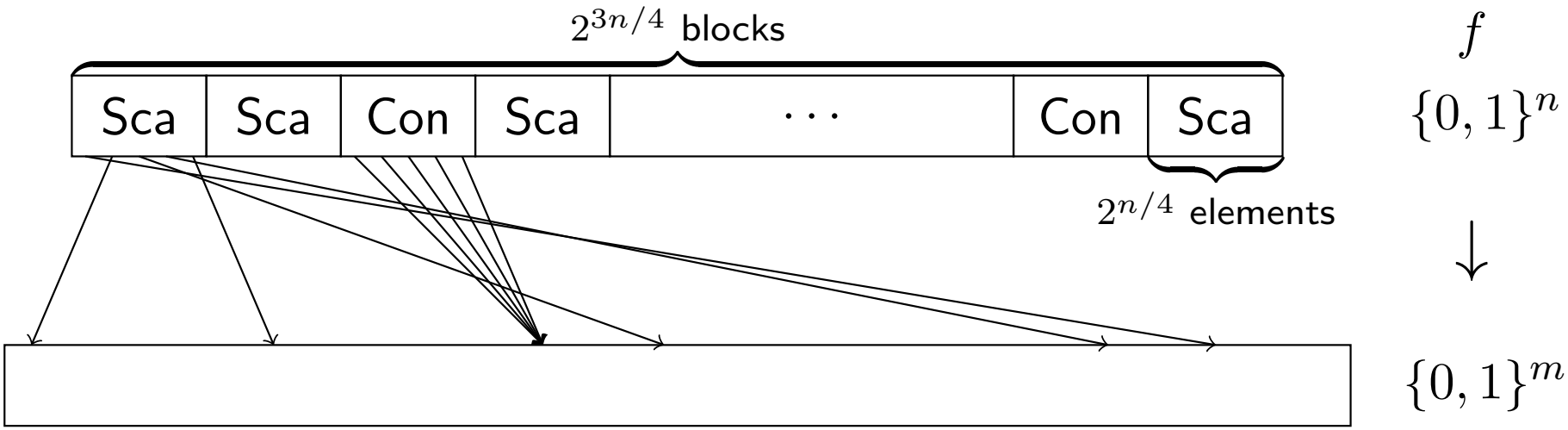
Def: SDU Algorithm

- $H_{\mathsf{sh}}(f(U_n)) \geq \tau + 1 \quad \Rightarrow \quad d_{\mathrm{TV}}(A^f(U_{n'}), U_{m'}) < \varepsilon.$
- $H_{\mathsf{sh}}(f(U_n)) \leq \tau - 1 \quad \Rightarrow \quad \mathrm{Supp}\left(A^f(U_{n'})\right)/2^{m'} \leq \varepsilon.$

1. Construct distributions $\mathcal{D}_H$ and $\mathcal{D}_L$:
   - Sample f from $\mathcal{D}_H$, then $H_{\mathsf{sh}}(f(U_n)) \geq \tau + 1$ w.h.p.
   - Sample f from $\mathcal{D}_L$, then $H_{\mathsf{sh}}(f(U_n)) \leq \tau - 1$ w.h.p.
2. A cannot "behave very different" on both distributions by making only $q = o(n^2)$ queries.

Construction of f

- Partition the domain into s blocks, each with t elements ($s \cdot t = 2^n$)
  - Concentrated: map to the same element.
  - Scattered: map to all distinct elements.



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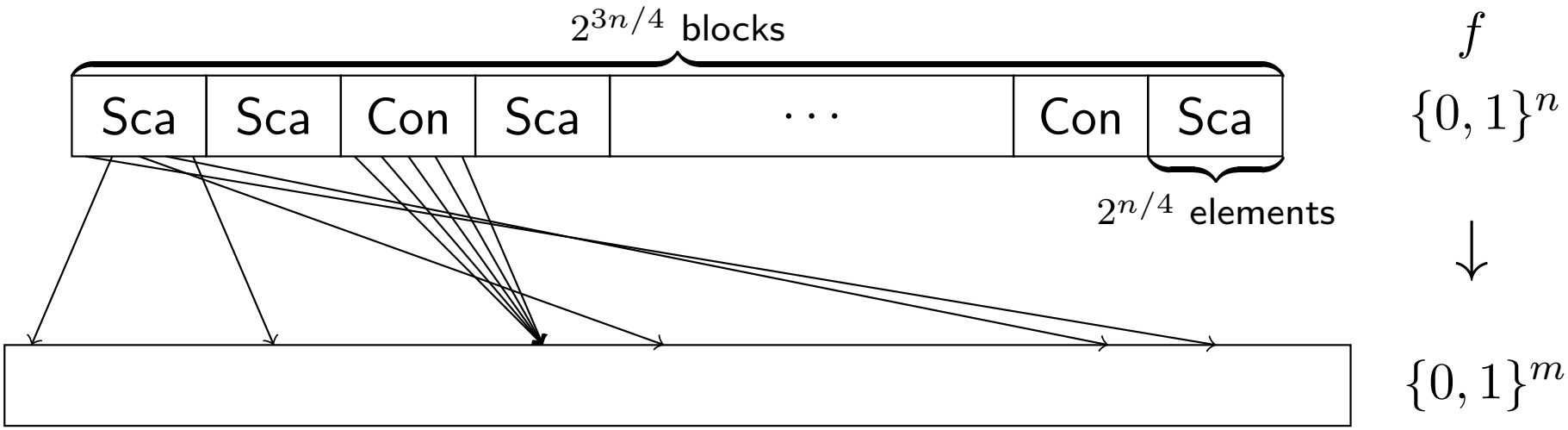
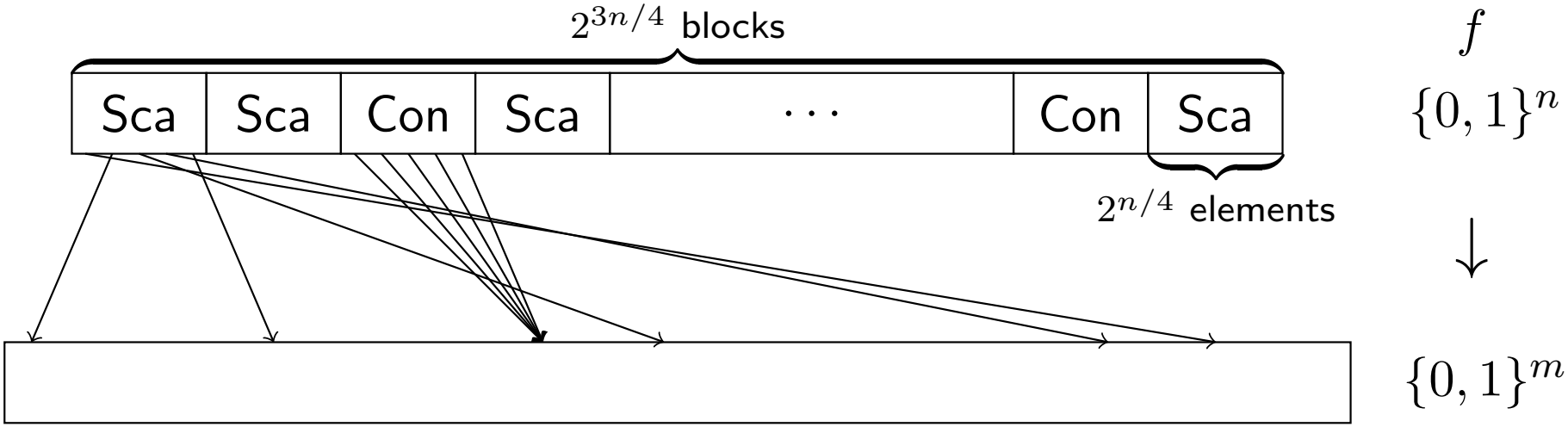


- $\geq s \cdot (1/2 + 4/n)$ blocks are scattered $\Rightarrow$ $H_{\mathsf{sh}}(f) \geq 7n/8 + 1$
- $\leq s \cdot (1/2 - 4/n)$ blocks are scattered $\Rightarrow$ $H_{\mathsf{sh}}(f) \leq 7n/8 - 1$

$\mathcal{D}_H$ and $\mathcal{D}_L$



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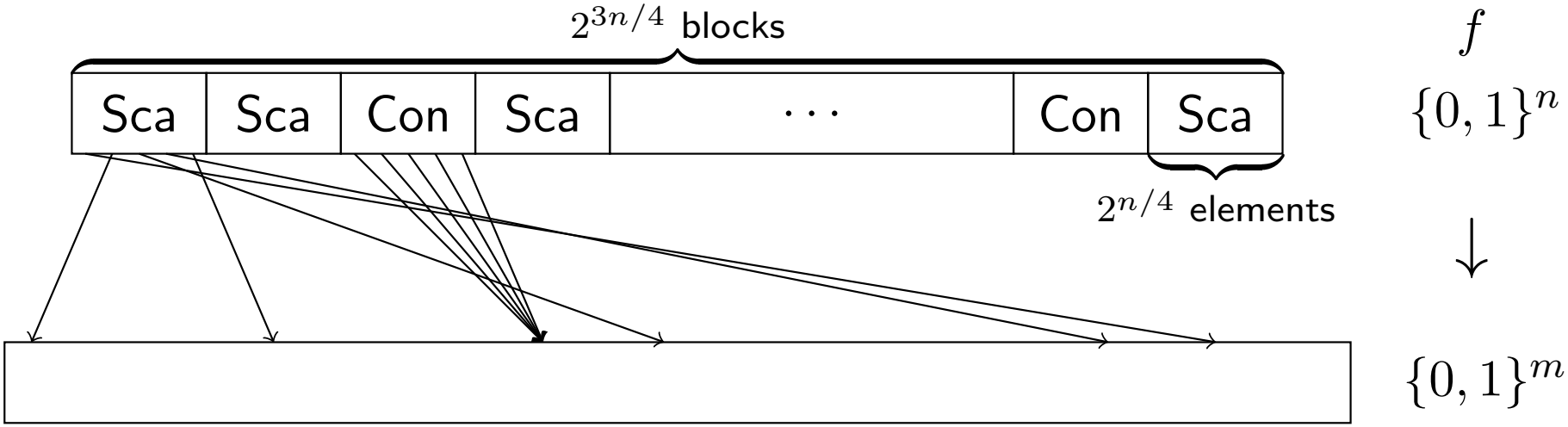

1. Randomly partition $\{0,1\}^n$ into $2^{3n/4}$ blocks.
2. Decide each block to be scattered or concentrated.
   - $\mathcal{D}_H$: scattered with probability $(1/2 + 5/n)$,
     then w.h.p, $\geq s \cdot (1/2 + 4/n)$ blocks are scattered
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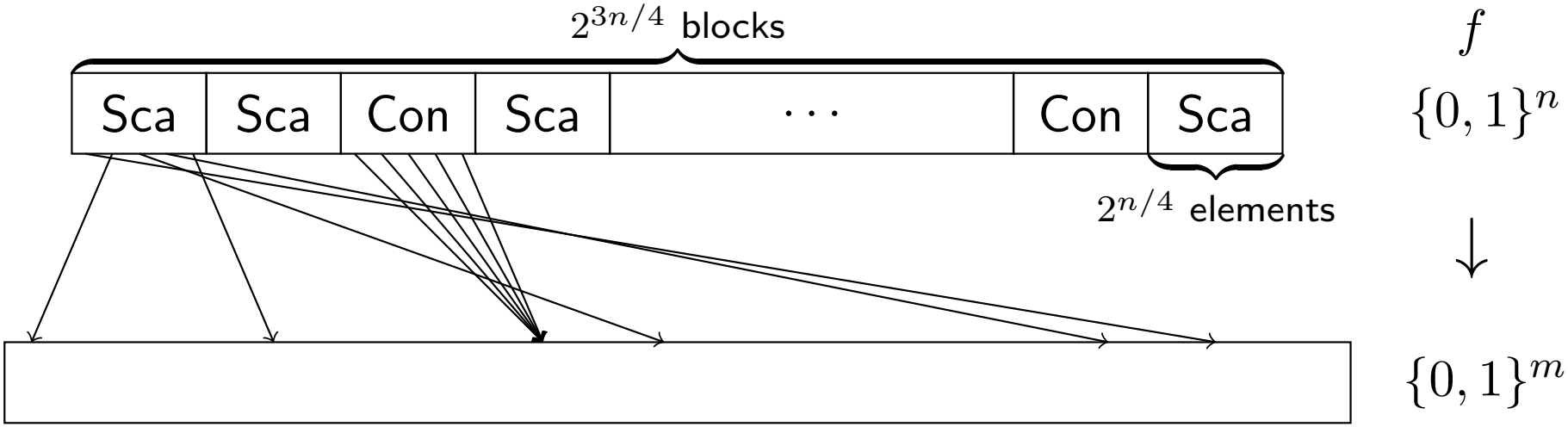

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3. Random mapping:
   - *Randomly* map each element in a *scattered* block.
   - Map all t elements in a *concentrated* block to a *random* target.

Intuitions for the Hard Distributions

Fix an SDU algorithm $A^{(\cdot)}$. An input w is **block-compatible** (B.C.) for f if each block is queried (when evaluating $A^f(w)$) at most once.

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Why random mapping?

- Conditioning on B.C., an algorithm cannot distinguish scattered or concentrated blocks.
- $O(n)$ queries is sufficient if the algorithm knows the block is scattered or concentrated!

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We will focus on the event $\left[\exists \text{B.C. } w, A^f(w) = z\right]$.

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Main Technical Lemma

Suppose A^f algorithm makes q oracle queries, for most $z \in \{0,1\}^{m'}$,

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which concludes that $q = \Omega(n^2)$ (or $\Omega(n^2 \log(1/\varepsilon))$.

Primitive Intuition for Distinguishing $\mathcal{D}_L$ and $\mathcal{D}_H$

We flip coins to decide each block is scattered or concentrated.

$$\mathcal{D}_H : \mathsf{Bern}(1/2 + 5/n) \qquad \mathcal{D}_L : \mathsf{Bern}(1/2 - 5/n)$$

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- Conditioning on the "balance" event, the ratio is at most

$$\begin{aligned}&(1 + 20/n)^{q\cdot(1/2+6/n)} \times (1 - 20/n)^{q\cdot(1/2-6/n)} \\ &\leq\ (1 + 20/n)^{12q/n} \times (1 - 20/n)^{-12q/n}\ =\ 2^{O(q/n^2)}.\end{aligned}$$

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- Warning! To distinguish two cases in "NISZK"-sense (instead of BPP) $O(n)$ queries are sufficient.

Comparison to [Lovett Zhang 17]

Entropy reversal: A has to make **exponentially** many queries such that

- $f(U_n)$ has high entropy $\Rightarrow$ $A^f(U_{n'})$ has small support.
- $f(U_n)$ has low entropy $\Rightarrow$ $A^f(U_{n'})$ is close to uniform

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Lemma (Lemma in [LZ17])

$$\Pr_{f\sim\mathcal{D}_H}\left[\exists B.C.\ w, A^f(w)=z\right] \geq \Pr_{f\sim\mathcal{D}_L}\left[\exists B.C.\ w, A^f(w)=z\right] + \mathrm{negl}$$

Lemma (This work)

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Technical Sketch

Let $\{w_1, \ldots, w_{2^{n'}}\} = \{0,1\}^{n'}$. $W_\ell = \{w_1, \ldots, w_\ell\}$.

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$$= \sum_\ell \Pr\left[\nexists w \in W_{\ell-1} \text{ s.t. } A^f(w) \neq z \mid A^f(w_\ell) = z\right] \times \Pr\left[A^f(w_\ell) = z\right]$$

$$= \sum_\ell \left(1 - \Pr\left[\exists w, \tilde{A}^f(w) = z \mid A^f(w_\ell) = z\right]\right) \times \Pr\left[A^f(w_\ell) = z\right]$$

where $\tilde{A}^f(w) = \begin{cases} A^f(w) & \text{if } w \in W_\ell \\ \perp & \text{Otherwise} \end{cases}$

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Thanks!