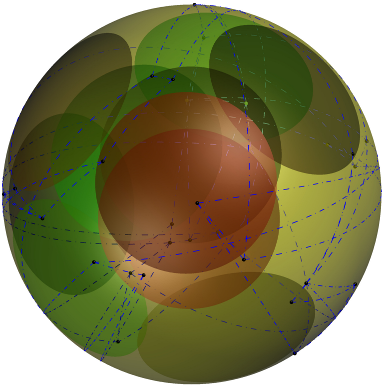
From Weak to Strong LP Gaps for all CSPs



Mrinalkanti Ghosh

joint work with:
Madhur Tulsiani

MAX k-CSP

- n variables
- m constraints

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$$x_3 \vee \overline{x}_9 \vee x_{23}$$
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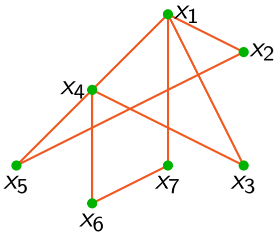

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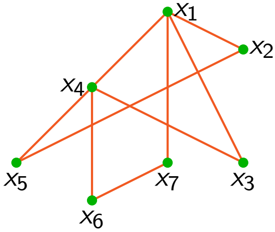

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Approximation Problem: Approximate the *fraction* of constraints simultaneously satisfiable.

MAX k-CSP

- n variables taking values in some finite domains.
- m constraints: each is a non-negative k-ary function.
- Satisfy as many as possible.

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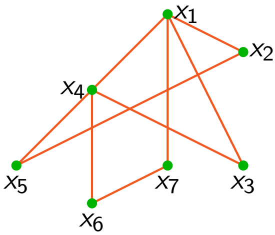

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CSPs and Relaxations

MAX k-CSP (f): for i-th constraint, let $S_{C_i} := (x_{i_1}, \cdots, x_{i_k})$. Then:

$$C_i \equiv f(x_{i_1} + b_{i_1}, \cdots, x_{i_k} + b_{i_k}) \equiv \sum_{\alpha \in \{0,1\}^{S_{C_i}}} f(\alpha + b_{C_i}) \cdot x_{(S_{C_i}, \alpha)},$$

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$$\sum_{\substack{\alpha \in \{0,1\}^{S_C} \\ \alpha(i)=b}} x_{(S_C,\alpha)} = x_{(i,b)} \qquad \forall C \in \Phi, i \in S_C, b \in \{0,1\}$$

$$\sum_{b \in \{0,1\}} x_{(i,b)} = 1 \qquad \forall i \in [n]$$

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$$x_{(S,\alpha)} \geq 0$$

#constraints = $\Theta\left(m \cdot 2^k\right)$

Extended Formulation and Sherali-Adams Relaxation

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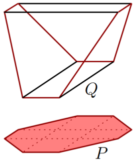

Image from
[Fiorini-Rothvoss-Tiwari-11]

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- Optimize objective $\langle w_\Phi, x \rangle$ (depending on Φ) over P.
- Introduce additional variables y. Optimize over polytope $P = \{x \mid \exists y \;\; Ex + Fy = g, y \geq 0\}$. Size equals #variables + #constraints.

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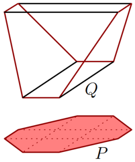

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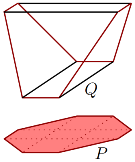

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- Variables: $x_{(S,\alpha)}$, $|S| \leq t$, $\alpha \in \{0,1\}^S$.

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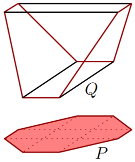
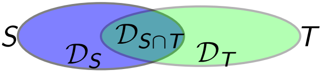


SA:



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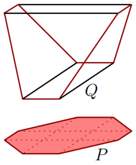

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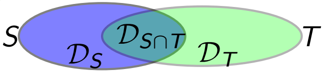
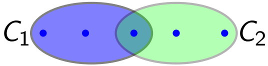


Basic:



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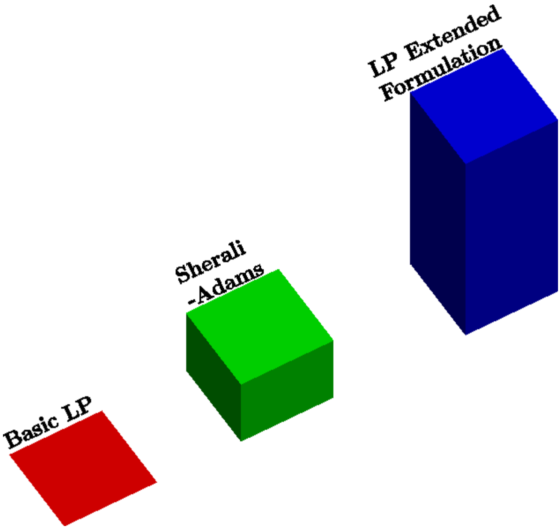
- Similarly, for Basic LP solution.

Result



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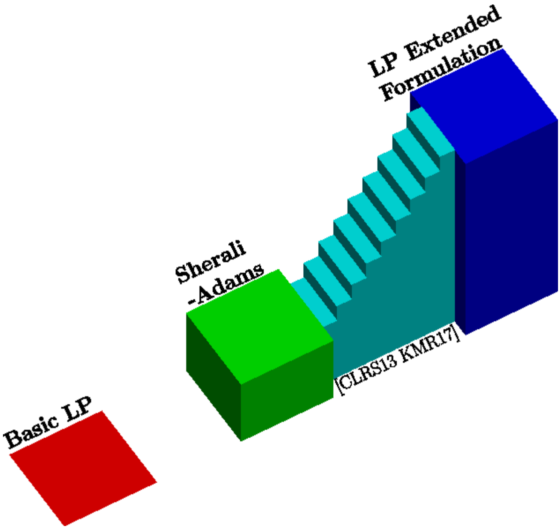
LP Extended
Formulation
Sherali
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[CLRS13 KMR17]
Basic LP

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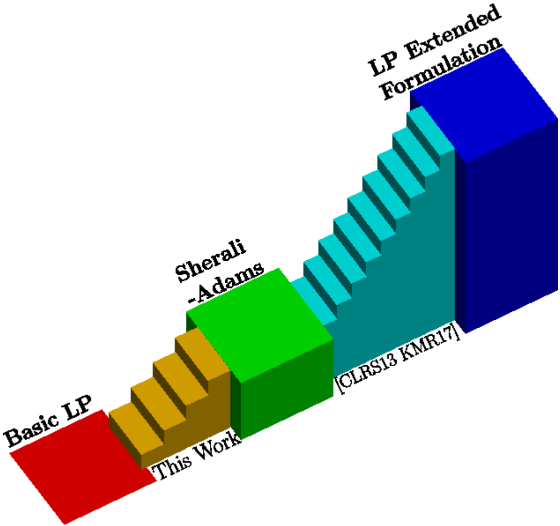
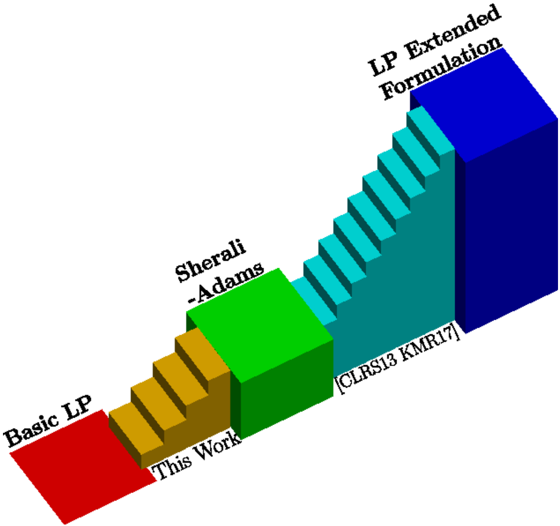


Main Theorem: For *all* CSPs, if Basic LP has integrality gap of (c, s) then for all $\varepsilon > 0$, there exist large enough instance(s) with integrality gap of $(c - \varepsilon, s + \varepsilon)$ for $SA(\widetilde{O}_{\varepsilon}(\log n))$.

Result

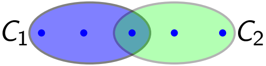


With [Kothari-Meka-Raghavendra-17]: **For *all* CSPs, if Basic LP has (c, s) gap, then so does any LP Extended Formulation of size $n^{\widetilde{O}(\log n)}$.**

Ignoring ε losses.

Hard Instance

Basic:



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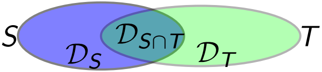

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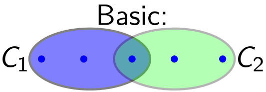

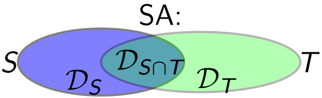

Use the hard instance Φ_0 of the basic relaxation as template to build the new hard instance on n variables and $m = \Delta \cdot n$ constraints.

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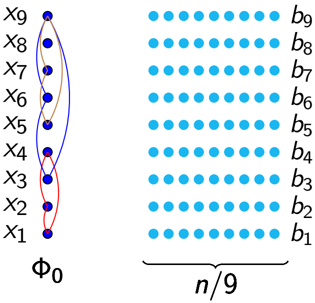

- For each variable in Φ_0, create bucket with large number of variables.

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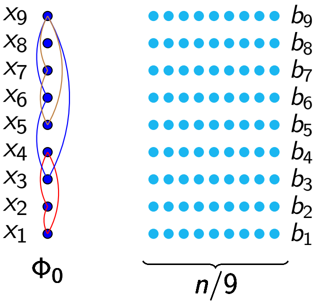

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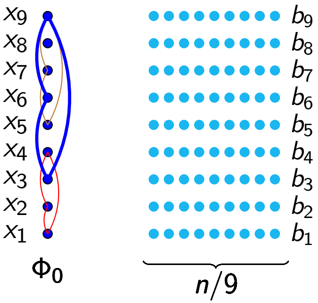

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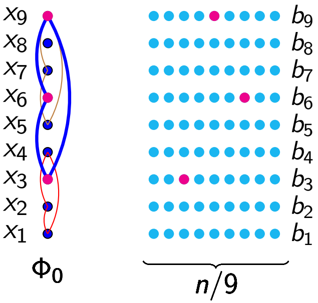

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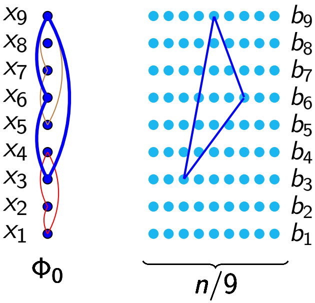

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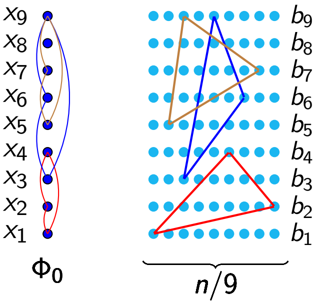

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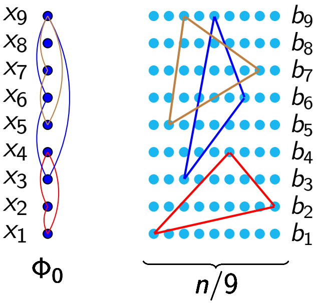

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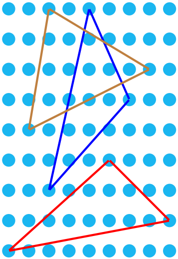
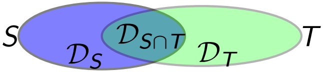
Overview - Completeness

Instance:



Consistent Distributions:



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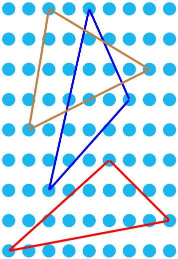

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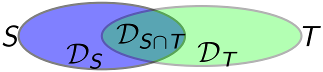

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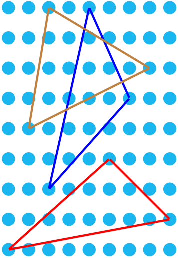

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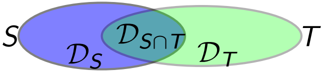

Step 1: Consistent Low-Diameter Decompositions.

Step 2: Construction of consistent distribution – Conditioning and propagating.

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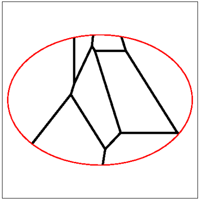

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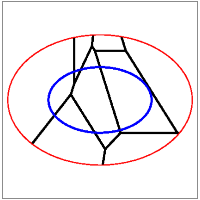

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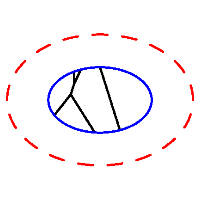

Figure: $S \subset T$

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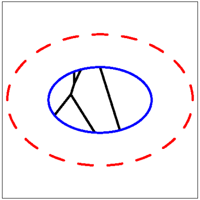

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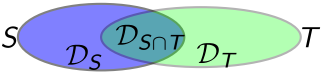

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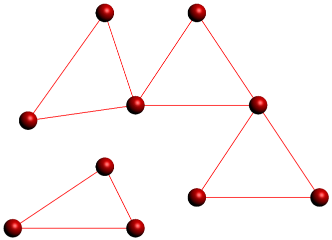
Step 2: Conditioning and Propagation

Assume: $c = 1$



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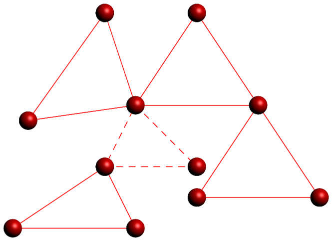

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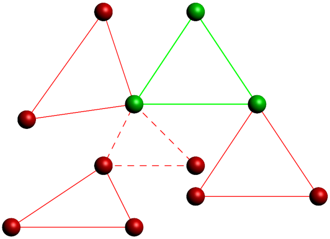

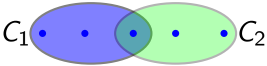

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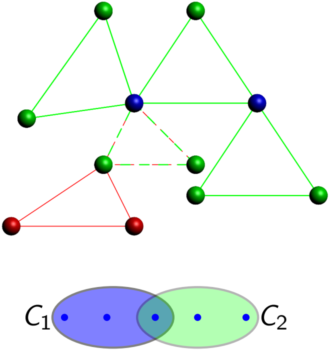

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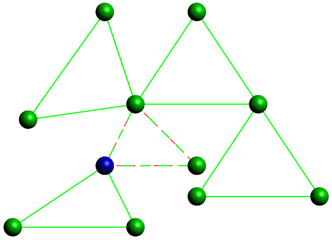

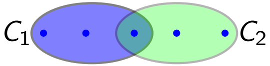

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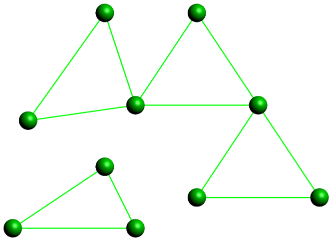

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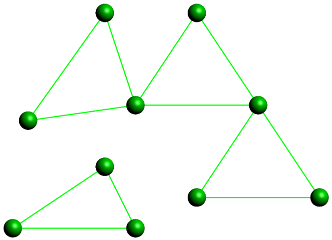

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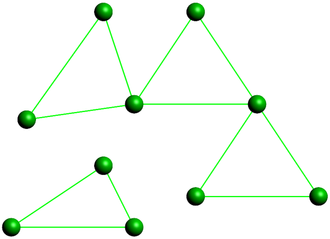

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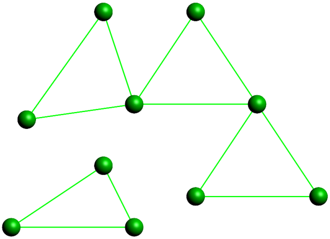

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High girth + consistent low-diameter decomposition $\Rightarrow$ Consistent Distribution.

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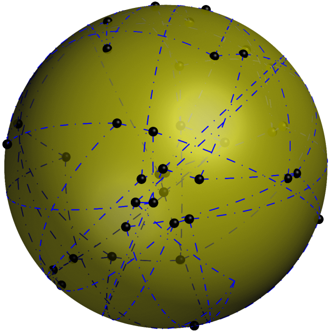

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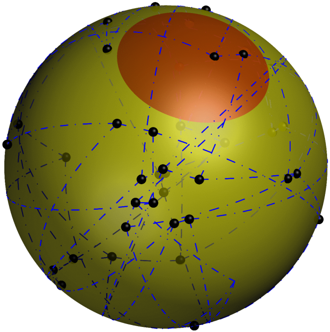

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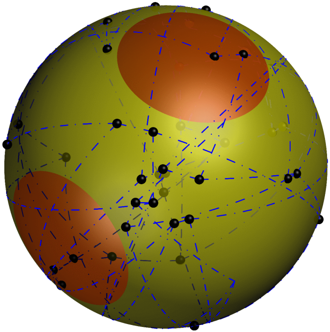

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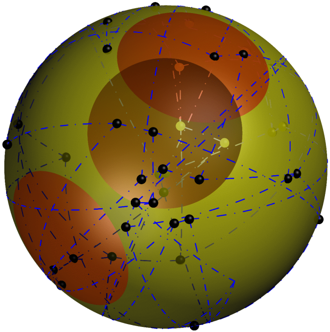

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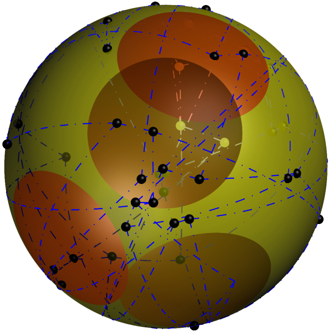

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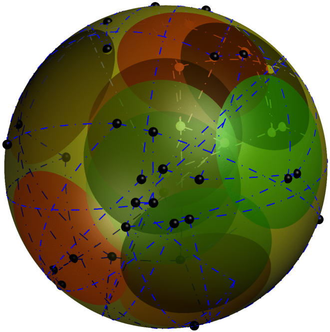

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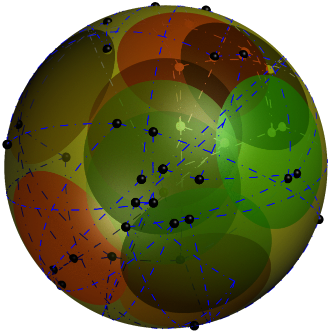

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The probability of cutting a hyperedge dictates the size of the sets we can handle.

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Questions?

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